Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | |

Course Code | 2 | 2 | M | D | A | 3 | 1 |

Third Semester B.E. Degree Examinations, March/April 2024

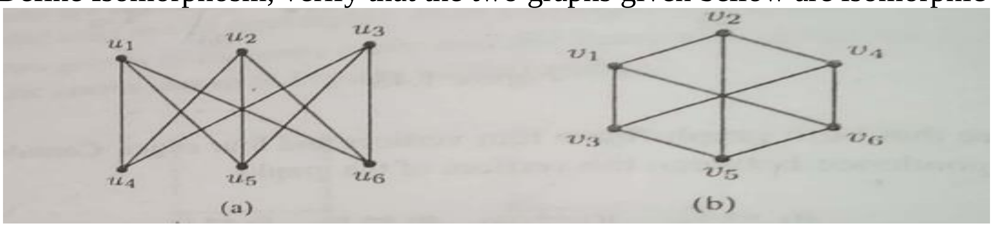
Graph Theory and Discrete Mathematical Structures, Probability and Statistics

AIML, CSE (AI) and CSE (DS)

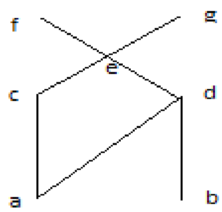
Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any FIVE full questions choosing ONE Full Question from each Module***
2. Formula Handbook is permitted
3. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		Module-1		
1.	**a.**	Define with examples (i) simple graph (ii) complete graph (iii) Bipartite graph.	**06**	(2 :1 : 1.2.1)
	b.	If $G = G(V, E)$ is a simple graph, prove that $2\|E\| \le \|V\|^2 - \|V\|$.	**07**	(2:1 : 1.2.1)
	c.	Prove that in every graph, the number of vertices of odd degrees is even.	**07**	(2 :1 : 1.2.1)
		(OR)		
2.	**a.**	For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \le \frac{2m}{n} \le \Delta$.	**06**	(2 :1 : 1.2.1)
	b.	In the complete graph with n vertices, where n is an odd number ≥ 3, there are $(n-1)/2$ edge-disjoint Hamiltonian cycles.	**07**	(2:1 : 1.2.1)
	c.	Define Isomorphosm, verify that the two graphs given bellow are isomorphic	**07**	(2 :1 : 1.2.1)



Q. No		Question	Marks	(RBTL:CO:PI)
		Module-2		
3.	**a.**	Let $A = \{1, 2, 3, 4\}$ and let R be the relation on A defined by xRy if and only if "x divides y", written $x \mid y$. (i) Write down R as a set of ordered pairs (ii) Draw the digraph of R (iii) Determine the in-degrees and out-degrees of the vertices in the digraph	**06**	(2 :2 : 1.2.1)
	b.	Draw the Hasse diagram representing the positive divisors of 36.	**07**	(2:2 : 1.2.1)
	c.	Consider the poset whose Hasse diagram is shown below. Find LUB and GLB of $B = \{c, d, e\}$	**07**	(2 :2 : 1.2.1)



(OR)

4. a. Consider the functions f and g defined by $f(x)=x^3$ and $g(x)=x^2+1, \forall x \in R$. Find gof, fog, f^2 and g^2. **06** (2 :2 : 1.2.1)

b. Let $p=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$. **07** (2:2 : 1.2.1)

(a) Write p as a product of disjoint cycles (b) Compute p^{-1} (c) Compute p^2 and p^3 (d) Find the smallest positive integer k such that $p^k=I_A$

c. Let $f:A \to B, g:B \to C$ and $h:C \to D$ be three functions. Prove that $(hog)of=ho(gof)$. **07** (2 :2 : 1.2.1)

Module-3

5. a. Solve the recurrence relation $a_n=7a_{n-1}$, where $n \ge 1$, given that $a_2=98$. **06** (2 :3 : 1.2.1)

b. Solve the recurrence relation $2a_n-3a_{n-1}=0$, for $n \ge 1$, given that $a_4=81$. **07** (2:3 : 1.2.1)

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **07** (2 :3 : 1.2.1)

(OR)

6. a. Solve the recurrence relation $F_{n+2}=F_{n+1}+F_n$ for $n \ge 0$, given $F_0=0, F_1=1$. **06** (2 :3 : 1.2.1)

b. Solve the recurrence relation $a_{n+2}+4a_{n+1}+4a_n=7, n \ge 0.$ given that $a_0=1,\ a_1=2.$ **07** (2:3 : 1.2.1)

c. Solve the recurrence relation $a_{n+2}-10a_{n+1}+21a_n=3n^2-2, n \ge 0.$ **07** (2 :3 : 1.2.1)

Module-4

7. a. Find the correlation coefficient and the equation of the lines of regression for the following values of x and y. **06** (2 :4 : 1.2.1)

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

b. If θ is the acute angle between the lines of regression, then show that **07** (2:4 : 1.2.1)

$$\tan\theta=\frac{\sigma_x\sigma_y}{\sigma_x{}^2+\sigma_y{}^2}\left(\frac{1-x^2}{r}\right)$$

c. Ten competitors in a beauty contest are ranked by two judges in the following order. Compute the coefficient of rank correlation. **07** (2 :4 : 1.2.1)

I	1	6	5	3	10	2	4	9	7	8
II	6	4	9	8	1	2	3	10	5	7

(OR)

8. a. Find the equation of the best fitting straight line for the following data and hence estimate the value of the dependent variable corresponding to the value 30 of the independent variable **06** (2 :4 : 1.2.1)

x	5	10	15	20	25
y	16	19	23	26	30

b Fit a parabola $y=ax^2+bx+c$ for the following data: **07** (2:4 : 1.2.1)

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

c. Fit a best fitting curve in the form $y = ax^b$ for the following data:		**07**	(2 :4 : 1.2.1)

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

Module-5

9. a. Find the value of k such that the following distribution represents finite probability distribution. Hence find its mean and standard deviation. Also find $P(x \le 1), P(x \succ 1), P(-1 < x \le 2)$ **06** (2 :5 : 1.2.1)

x	-3	-2	-1	0	1	2	3
$P(x)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

b. Find the mean and standard deviation of Binomial distribution. **07** (2:5: 1.2.1)

c. When a coin is tossed 4 times, find the probability of getting (i) exactly one head (ii) at most 3 heads (iii) at least 2 heads. **07** (2 :5 : 1.2.1)

(OR)

10 a. The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 to 75. **06** (2 :5 : 1.2.1)

b. The joint distribution table for two random variables X and Y is as follows **07** (2:5: 1.2.1)

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal distribution of X and Y.
Also compute
(i) Expectations of X and Y (ii) S.Ds of X and Y (iii) Covariance of X and Y (iv) Correlation of X and Y
Further verify that X and Y are dependent random variables.

c. Suppose X and Y are independent random variables with the following respective distribution, find the joint distribution of X and Y. Also verify that $C0V(X,Y) = 0$. **07** (2 :5 : 1.2.1)

x_i	1	2
$f(x_i)$	0.7	0.3

y_i	-2	5	8
$f(y_i)$	0.3	0.5	0.2

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