Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 2 | M | C | S | 3 | 1 |

Third Semester B.E. Degree Examinations, March/April 2024

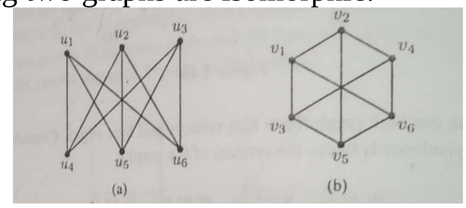
GRAPH THEORY & DISCRETE MATHEMATICAL STRUCTURES

(Computer Science & Engineering)

Duration: 3 hrs **Max. Marks: 100**

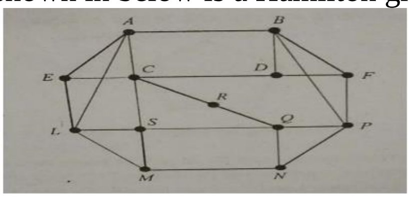
Note:* *1. Answer any* FIVE *full questions choosing ONE Full Question from each Module
2. Formula Handbook is permitted
3. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		Module-1		
1.	**a.**	Show that a complete graph with n vertices, namely K_n, has $\frac{1}{2}n(n-1)$ edges.	**06**	(2 :1 : 1.2.1)
	b.	For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \le \frac{2m}{n} \le \Delta$.	**07**	(2 :1 : 1.2.1)
	c.	Show that the following two graphs are isomorphic:	**07**	(2 :1 : 1.2.1)



(OR)

Q. No		Question	Marks	(RBTL:CO:PI)
2.	**a.**	Show that the graph shown in below is a Hamilton graph.	**6**	(2 :1 : 1.2.1)



Q. No		Question	Marks	(RBTL:CO:PI)
	b.	Exhibit the following (i) A graph which has both an Euler circuit and Hamilton cycle. (ii) A graph which has an Euler circuit but no Hamilton cycle. (iii) A graph which has a Hamilton cycle but no Euler circuit. *(iv)* A graph which has neither a Hamilton cycle nor an Euler circuit.	**7**	(2 :1 : 1.2.1)
	c.	In the complete graph with n vertices, where n is an odd number ≥ 3, there are $(n-1)/2$ edge-disjoint Hamiltonian cycles.	**7**	(2 :1 : 1.2.1)
		Module-2		
3.	**a.**	Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees. If $\|E_1\|=19$ and $\|V_2\|=3\|V_1\|$, determine $\|V_1\|$, $\|V_2\|$ and $\|E_2\|$.	**6**	(2 :2 : 1.2.1)
	b.	Prove that, any connected graph with n vertices and n-1 edges is a tree.	**7**	(2 :2 : 1.2.1)
	c.	Suppose that a tree T has N_1 vertices of degree 2, N_2 vertices of degree 2, N_3 vertices of degree 3,…. , N_k vertices of degree k. Prove that $N_1 = 2+ N_3+2N_4+3N_5+....+ (k-2) N_k$.	**7**	(2 :2 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

(OR)

4. **a.** A code for $\{a,b,c,d,e\}$ is given by $a:00,\ b:01,\ c:101,\ d:x10,\ e:yz1$. Where $x,y,z \in \{0,1\}$. Determine x, y and z so that the given code is a prefix code. **06** (2 :2 : 1.2.1)

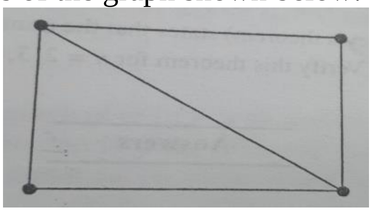
b. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. **07** (2 :2 : 1.2.1)

c. Construct an optimal prefix code for the symbols with given weights in the following table. **07** (2 :2 : 1.2.1)

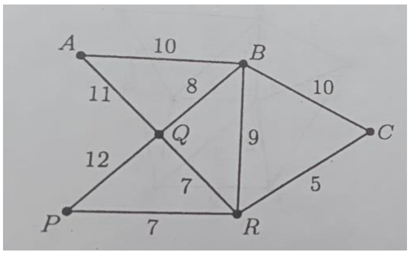
Symbols:	a	b	c	d	e	f	g
Weight:	10	30	5	15	20	15	5

Module-3

5. **a.** Find all the spanning trees of the graph shown below: **06** (2 :3 : 1.2.1)



b. Using *Kruskal's* algorithm, find a minimal spanning tree for the weighted graph shown below: **07** (2 :3 : 1.2.1)



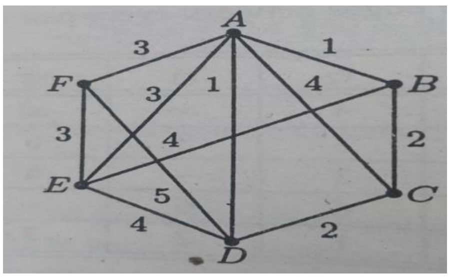
c. Eight cities A, B, C, D, E, F, G, H are required to be connected by a new railway network. The possible tracks and the cost of involved to lay them (in crores of rupees) are summarized in the following table: **07** (2 :3 : 1.2.1)

Track between	Cost	Track between	Cost
A and B	155	D and F	100
A and D	145	E and F	150
A and G	120	F and G	140
B and C	145	F and H	150
C and D	150	G and H	160
C and E	95		

Determine a railway network of minimal cost that connects all these cities.

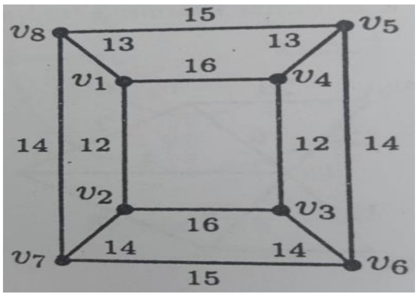
(OR)

6. **a.** Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below. **06** (2 :3 : 1.2.1)

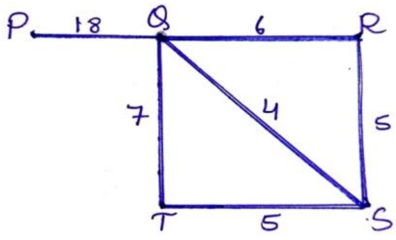


Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

b. Find (by Kruskal's and Prim's method) a minimal spanning tree for the following weighted graph: **07** (2 :3 : 1.2.1)



c. Find the maximum flow possible between the vertices P and S in the network shown in below figure **07** (2 :3 : 1.2.1)



Module-4

7. a. Let $A=\{1,2,3,4\}$ and let R be the relation on A defined by xRy if and only if "x divides y", written $x \text{ I } y$. (i) Write down R as a set of ordered pairs (ii) Draw the digraph of R (iii) Determine the in-degrees and out-degrees of the vertices in the digraph **06** (2 :4 : 1.2.1)

b. Construct the digraph of R and list the in-degrees and out-degrees of all vertices, given $A=\{a,b,c,d\}$ and R be a relation on A that has the matrix **07** (2 :4 : 1.2.1)

$$M_R=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}$$

c. Draw the Hasse diagram representing the positive divisors of 36. **07** (2 :4 : 1.2.1)

(OR)

8. a. Let f and g be functions from R to R defined by $f(x)=ax+b$ and $g(x)=1-x+x^2$. If $(gof)(x)=9x^2 9x+3$, determine a and b. **06** (2 :4 : 1.2.1)

b. Let $p=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 4 & 3 & 1 & 5 & 6\end{pmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$ **07** (2 :4 : 1.2.1)

(i) Write p as a product of disjoint cycles. (ii) Compute p^{-1}

(iii) Compute p^2 and p^3 (iv) Find the smallest positive integer k such that $p^k=I_A$

c. ABC is an equilateral triangle whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm. **07** (2 :4 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

Module-5

9.	**a.**	The number of virus affected files in a system is 1000 (to start with) and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.	**06**	(2 :5 : 1.2.1)
	b.	Solve the recurrence relation $2a_n - 3a_{n-1} = 0$, for $n \geq 1$, given that $a_4 = 81$.	**07**	(2 :5 : 1.2.1)
	c.	Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$, for $n \geq 2$, given that $a_0 = 1, a_1 = 2$.	**07**	(2 :5 : 1.2.1)

(OR)

10	**a.**	Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given $F_0 = 0, F_1 = 1$.	**06**	(2 :5 : 1.2.1)
	b.	Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 4a_n = 7$, $n \geq 0$. given that $a_0 = 1$, $a_1 = 2$.	**07**	(2 :5 : 1.2.1)
	c.	Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2$, for $n \geq 0$.	**07**	(2 :5 : 1.2.1)

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Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)