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Course Code

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Third Semester B.E. Degree Examinations, April/May 2023
TRANSFORM CALCULUS AND NUMERICAL METHODS
 (Common to ECE & EEE)

Duration: 3 hrs

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. Missing data, if any, may be suitably assumed

<u>O. No</u>	<u>Question</u>	<u>Marks</u>	<u>(RBTL:CO:PI)</u>
MODULE – 1			
1.	a. State and prove Euler's formulae in Fourier series.	06	(3 : 1 : 1.2.1)
	b. Obtain the Fourier series for the function $f(x) = \begin{cases} -\pi & \text{in } -\pi < x < 0 \\ x & \text{in } 0 < x < \pi \end{cases}$	07	(2 : 1 : 1.2.1)
	c. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$	07	(2 : 1 : 1.2.1)
OR			
2.	a. Find the Fourier series of the periodic function $f(x) = 2x - x^2$ in $0 < x < 3$	06	(2 : 1 : 1.2.1)
	b. Obtain the Sine half range Fourier series of $f(x) = x^2$ in $0 < x < \pi$	07	(2 : 1 : 1.2.1)
	c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data.	07	(3 : 1 : 1.2.1)

x^0	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

MODULE – 2

3.	a. Find the Complex Fourier transform of the function $f(x) = \begin{cases} 1 & \text{for } x \leq a \\ 0 & \text{for } x > a \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.	06	(2 : 2 : 1.2.1)
	b. Find the Fourier Transform of $f(x) = e^{- x }$	07	(2 : 2 : 1.2.1)
	c. If $f(x) = \begin{cases} 1-x^2, & x < 1 \\ 0, & x \geq 1 \end{cases}$ find the Fourier transform of $f(x)$ and hence find the value of $\int_0^\infty \frac{x \cos x - \sin x}{x^3} dx$	07	(2 : 2 : 1.2.1)

OR

4.	a. Find the Fourier Sine Transform of $f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$	06	(2 : 2 : 1.2.1)
	b. Find the Fourier Sine transform of $\frac{e^{-ax}}{x}, a > 0$	07	(2 : 2 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)

- c. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$

07 (2 : 2 : 1.2.1)

MODULE – 3

5. a. Obtain the Z-Transform of $\cos n\theta$ and $\sin n\theta$. **06** (2 : 3 : 1.2.1)
 b. Find the Z-Transform of $\sin(3n+5)$. **07** (2 : 3 : 1.2.1)
 c. Using $Z_T(n^2) = \frac{z^2 + z}{(z-1)^3}$ show that $Z_T((n+1)^2) = \frac{z^2 + z^2}{(z-1)^3}$ **07** (2 : 3 : 1.2.1)

OR

6. a. Find the Inverse Z-Transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$ **06** (2 : 3 : 1.2.1)
 b. Solve the Difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-Transform. **07** (2 : 3 : 1.2.1)
 c. Find the Z-Transform and ROC, For the signal **07** (2 : 3 : 1.2.1)

$$x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n]$$

MODULE – 4

7. a. Use Taylor's series method to find y at x = 0.1 considering terms up to the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$. **06** (2 : 4 : 1.2.1)
 b. Using Modified Euler's method find y(0.1) correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ taking h = 0.1 **07** (2 : 4 : 1.2.1)
 c. Use Runge-Kutta method of fourth order to find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking h = 0.2 **07** (2 : 4 : 1.2.1)

OR

8. a. Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at x = 0.8 by applying Milne's method. **06** (2 : 4 : 1.2.1)
 b. Apply Milne's method to compute y(1.4) correct to 4 decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data: **07** (2 : 4 : 1.2.1)

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

- c. Compute y(1.4) correct to 3 decimal places, given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ by applying Adams-Bashforth method given that

x	1	1.1	1.2	1.3
y	1	0.996	0.986	0.972

MODULE – 5

9. a. Obtain the Picard's second approximation to the solution of the system of equations $\frac{dx}{dt} = 2x + 3y, \frac{dy}{dt} = x - 3y, t=0, x=0, y=1/2$. Hence find x and y at $t = 0.2$ 06 (2 :5 : 1.2.1)
 b. Use fourth order R-K method to solve the system of equations: 07 (3 :5 : 1.2.1)
 $\frac{dx}{dt} = y - t, \frac{dy}{dt} = x + t, x = 1, y = 1$ at $t = 0$. Compute $x(0.1)$ and $y(0.1)$.
 c. Solve $\frac{dy}{dx} = 1 + zx, \frac{dz}{dx} + xy = 0, y(0) = 0, z(0) = 1$ at $x = 0.3$ by applying fourth order R-K method. 07 (3 :5 : 1.2.1)

OR

10. a. By R-K method of fourth order, solve $\frac{d^2y}{dx^2} = x \left(\frac{dy}{dx} \right)^2 - y^2$ for $x=0.2$ correct to four decimal places, using the initial conditions $y=1$ and $y' = 0$ when $x=0$. 06 (2 :5 : 1.2.1)
 b. Apply Milne's method to compute $y(1.4)$ given that $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ and the following table of initial values. 07 (3 :5 : 1.2.1)

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

- c. Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}$ and the following table of initial values. 07 (3 :5 : 1.2.1)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

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