Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 1 | M | C | M | 3 | 1 |

Third Semester B.E. Degree Examinations, April/May 2023

INTERGRAL TRANSFORMS AND NUMERICAL METHODS

(Common to ME & CV)

Duration: 3 hrs **Max. Marks: 100**

Note: 1. Answer any **FIVE** *full questions, choosing* **ONE** *full question from each module.*
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	**a.**	State and prove Euler's formulae in Fourier series.	**06**	(3 :1 : 1.2.1)
	b.	Find the Fourier series of $f(x)=x(2\pi-x)$ in $0<x<2\pi$.	**07**	(2 :1 : 1.2.1)
	c.	Express y as a Fourier Series up to the second harmonics for the following data.	**07**	(2 :1 : 1.2.1)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

OR

2.	**a.**	Obtain the Fourier series to represent $f(x)=\|x\|$ in $(-l,l)$.	**06**	(2 :1 : 1.2.1)
	b.	Obtain the Cosine half range Fourier series of $f(x)=x(\pi-x)$ in $0<x<\pi$.	**07**	(2 :1 : 1.2.1)
	c.	Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data.	**07**	(3 :1 : 1.2.1)

x	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

MODULE – 2

3.	**a.**	Find the Complex Fourier transform of the function. $f(x)=\begin{cases}1 & \text{for } \|x\|\le a\\ 0 & \text{for } \|x\|>a\end{cases}$	**06**	(2 :2 : 1.2.1)
	b.	Find the Fourier Transform of $f(x)=e^{-\|x\|}$.	**07**	(2 :2 : 1.2.1)
	c.	Find the Fourier Transform of $f(x)=\begin{cases}1-\|x\| & \text{for } \|x\|\le 1\\ 0 & \text{for } \|x\|>1\end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$.	**07**	(2 :2 : 1.2.1)

OR

4.	**a.**	Find the Fourier Sine and Cosine Transform of $f(x)=\begin{cases}x & 0<x<2\\ 0 & \text{elsewhere}\end{cases}$	**06**	(2 :2 : 1.2.1)
	b.	Find the Fourier Sine and Cosine Transform of $f(x)=e^{-\alpha x}, \alpha \succ 0$.	**07**	(2 :2 : 1.2.1)
	c.	Find the Fourier Cosine transform of $f(x)=\begin{cases}x & \text{for } 0<x<1\\ 2-x & \text{for } 1<x<2\\ 0 & \text{for } x>2\end{cases}$	**07**	(2 :2 : 1.2.1)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

MODULE – 3

5.	**a.**	Use Taylor's series method to find y at x = 0.1 considering terms up to the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$.	**06**	(2 :3 : 1.2.1)
	b.	Using modified Euler's method find y (0.1) correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2$, y (0) =1 taking h= 0.1.	**07**	(2 :3 : 1.2.1)
	c.	Solve: $(y^2 - x^2)dx = (y^2 + x^2)dy$ for x=0.2 given that y=1 at x=0 by applying R-K method of order 4.	**07**	(2 :3 : 1.2.1)

OR

6.	**a.**	Using Modified Euler's method find y (20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y (20) =5 and h=0.2.	**06**	(2 :3 : 1.2.1)
	b.	Apply Milne's method to compute y (1.4) correct to 4 decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data: $y(1) = 2,\ y(1.1) = 2.2156,\ y(1.2) = 2.4649,\ y(1.3) = 2.7514.$	**07**	(2 :3 : 1.2.1)
	c.	If $\frac{dy}{dx} = 2e^x - y, y(0) = 2,\ y(0.1) = 2.010,\ y(0.2) = 2.040,\ y(0.3) = 2.090.$ find y (0.4) by using Adams-Bashforth method.	**07**	(2 :3 : 1.2.1)

MODULE – 4

7.	**a.**	Use Picard's method to find y(0.1) and z(0.1) given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$, and $y(0) = 2, z(0) = 1$. (carry out two approximations)	**06**	(2 :4 : 1.2.1)
	b.	Solve $\frac{dy}{dx} = 1 + zx$, $\frac{dz}{dx} + xy = 0$, $y(0) = 0, z(0) = 1$ at $x = 0.3$ by applying fourth order R-K method.	**07**	(3 :4 : 1.2.1)
	c.	Obtain the Picard's third approximation to the solution of the system of equations $\frac{dx}{dt} = 2x + 3y$, $\frac{dy}{dt} = x - 3y$, t=0, x=0, y=1/2. Hence find x and y at t=0.2.	**07**	(3 :4 : 1.2.1)

OR

8.	**a.**	Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0.$ compute $y(0.2)$ and $y'(0.2)$ using R-K method of fourth order.	**06**	(2 :4 : 1.2.1)
	b.	Apply Milne's method to compute y (0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values.	**07**	(3 :4 : 1.2.1)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

	c.	By R-K method of fourth order, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x=0.2 correct to four decimal places, using the initial conditions y=1 and $y' = 0$ when x=0.	**07**	(3 :4 : 1.2.1)

MODULE – 5

			Marks	(RBTL : CO : PI)
9.	**a.**	Obtain the Z-Transform of $\cos n\theta$ and $\sin n\theta$.	**06**	(2 :5 : 1.2.1)
	b.	Find the Z-Transform of $\cosh\left(n\pi/2+\theta\right)$.	**07**	(2 :5 : 1.2.1)
	c.	Solve by using Z-Transforms: $y_{n+2}-4y_n=0$, given that $y_0=0, y_1=2$.	**07**	(3 :5 : 1.2.1)
		OR		
10.	**a.**	State and Prove Euler's Equation.	**06**	(2 :5 : 1.2.1)
	b.	Find the Extremal of the functional $\int_{x_1}^{x_2}\left(y' + x^2 y'^2\right)dx$	**07**	(2 :5 : 1.2.1)
	c.	Prove that the geodesics on a plane are straight lines.	**07**	(3 :5 : 1.2.1)

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Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**