Basavarajeswari Group of Institutions
BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT
(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 1 | M | A | T | 1 | 1 |

First Semester B.E. Degree Make-up Examinations, August 2022

CALCULUS AND LINEAR ALGEBRA

(Common to all Branches)

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	**a.**	With usual notation, prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$.	**06**	(2 :1 : 1.1.1)
	b.	Find the radius of curvature of the curve $r = a(1+\cos\theta)$, hence prove that $\frac{\rho^2}{r}$ is a constant.	**07**	(2 :1 : 1.1.1)
	c.	Find the radius of curvature for the Folium of De-Cartes $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it.	**07**	(1 :1 : 1.1.1)
		(OR)		
2.	**a.**	Find the angle of intersection of the curve $r = a(1-\sin\theta)$ and $r = a(1+\sin\theta)$.	**06**	(1 :1 : 1.1.1)
	b.	Show that for the curve $r(1-\cos\theta) = 2a$, ρ^2 varies as r^3.	**07**	(2 :1 : 1.1.1)
	c.	Find the center of curvature and circle of curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.	**07**	(2 :1 : 1.1.1)
		MODULE - 2		
3.	**a.**	Expand $e^{\sin x}$ as Maclaurin's series upto the term containing x^4.	**06**	(2 :2 : 1.1.1)
	b.	If $z = e^{ax+by} f(ax - by)$, prove that $b\left(\frac{\partial z}{\partial x}\right) + a\left(\frac{\partial z}{\partial y}\right) = 2abz$.	**07**	(2 :2 : 1.1.1)
	c.	Examine the function $xy(a - x - y)$ for extreme values.	**07**	(2 :2 : 1.1.1)
		(OR)		
4.	**a.**	Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \cdots$	**06**	(2 :2 : 1.1.1)
	b.	If $u = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$, prove that $u_{xx} + u_{yy} + u_{zz} = \frac{2}{u}$.	**07**	(2 :2 : 1.1.1)

(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)

	c.	Find the Jacobin of u, v, w with respect to x, y, z where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x + y + z$.	**07**	(2 :2 : 1.1.1)

MODULE-3

5.	**a.**	Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$.	**06**	(1 :3 : 1.1.1)
	b.	Change the order of integration and hence evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy\, dy\, dx$.	**07**	(1 :3 : 1.1.1)
	c.	Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.	**07**	(2 :3 : 1.1.1)

(OR)

6.	**a.**	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$ by changing the order of integration.	**06**	(1 :3 : 1.1.1)
	b.	Find the volume of the solid bounded by the planes $x = 0,\ y = 0,\ z = 0,\ x + y + z = 1$.	**07**	(1 :3 : 1.1.1)
	c.	Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$ by expressing in terms of Gama function.	**07**	(2 :3 : 1.1.1)

MODULE-4

7.	**a.**	Solve $\left[y\left(1+\frac{1}{x}\right) + \cos y \right] dx + \left[x + \log x - x\sin y \right] dy = 0$.	**06**	(1 :4 : 1.1.1)
	b.	If the temperature of the air is 30^0 C and the metal ball cools from 100^0 C to 70^0 C in 15 minutes, find how long it will take for the metal ball to reach a temperature of 40^0 C.	**07**	(1 :4 : 1.1.2)
	c.	Show that the family of curves $y^2 = 4a(x+a)$ is self orthogonal.	**07**	(2 :4 : 1.1.1)

(OR)

8.	**a.**	Solve: $x\frac{dy}{dx} + y = x^3 y^6$.	**06**	(1 :4 : 1.1.1)
	b.	Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$.	**07**	(1 :4 : 1.1.1)
	c.	An inductance 2 H and resistance 20 Ω are connected in series with e.m.f E volts (V). If the current is initially zero, when $t = 0$, find the current at the end of 0.01 seconds if $E = 100\ V$.	**07**	(2 :4 : 1.1.2)

(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)

MODULE-5

9.	**a.**	Find the rank of the matrix by elementary row transformation. $$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$	**06**	(1 :5 : 1.1.1)
	b.	Solve the following system of equations by Gauss-Seidel method. $10x+y+z=12,\ 2x+10y+z=13,\ 2x+2y+10z=14$	**07**	(2 :5 : 1.1.1)
	c.	Find the largest Eigen value and the corresponding Eigen vector of the matrix A by the power method given that $X=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ $$A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$	**07**	(1 :5 : 1.1.1)

(OR)

10.	**a.**	Solve the following system of equations by Gauss elimination method**.** $x+y+z=9,\ x-2y+3z=8,\ 2x+y-z=3$	**06**	(1 :5 : 1.1.1)
	b.	Test for consistency and solve: $x+2y+2z=1,\ 2x+y+z=2,\ 3x+2y+2z=3,\ y+z=0$	**07**	(2 :5 : 1.1.1)
	c.	Reduce the matrix $A=\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form.	**07**	(1 :5 : 1.1.1)

** ** **

(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)