

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

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Course Code

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Second Semester B.E. Degree Examinations, September/October 2023 Mathematics for MECHANICAL Stream- II

Duration: 3 hrs

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Missing data, if any, may be suitably assumed

<u>Q. No</u>	<u>Question</u>	<u>Marks</u>	<u>(RBTL:CO:PI)</u>
MODULE – 1			
1.	a. Find the directional derivative of the function $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along $2i - 3j + 6k$	06	1:1:1.2.1
	b. Show that $\vec{F} \cdot \text{curl } \vec{F} = 0$, if $\vec{F} = (x + y + 1)i + j - (x + y)k$,	07	3:1:1.2.1
	c. Find constants a and b such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$	07	1:1:1.2.1
OR			
2.	a. Find the total work done by the force represented by $\vec{F} = 3xyi - yj + 2zxk$ in moving a particle round the circle $x^2 + y^2 = 4$	06	1:1:1.2.1
	b. Verify Green's theorem in a plane for $\int (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$	07	3:1:1.2.1
	c. Verify Stoke's theorem for $\vec{F} = (2x - y)i - yz^2j - y^2zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, C is its boundary.	07	3:1:1.2.1
MODULE – 2			
3.	a. Solve: $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$	06	3:2:1.2.1
	b. Solve: $(D^2 - 3D + 2)y = \sin 3x$	07	3:2:1.2.1
	c. Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$	07	3:2:1.2.1
OR			
4.	a. Solve: $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters.	06	3:2:1.2.1
	b. Solve: $(x + 1)^2 \frac{d^2y}{dx^2} + (x + 1) \frac{dy}{dx} + y = 2\sin [\log(x + 1)]$	07	3:2:1.2.1
	c. Solve: $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$	07	3:2:1.2.1

MODULE – 3

5. a. Form the PDE by eliminating the arbitrary constants of 06 3:3:1.2.1
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
- b. Form the PDE by eliminating the arbitrary functions of 07 3:3:1.2.1
 $\phi(x + y + z, x^2 + y^2 - z^2) = 0$
- c. Find the PDE of the family of all spheres whose centres lie on the 07 1:3:1.2.1
plane $z = 0$ and have a constant radius ‘r’
- OR**
6. a. Solve: $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ given that $u = 0$ when $\frac{\partial u}{\partial t} = 0$ at $x = 0$ 06 3:3:1.2.1
- b. Solve: $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0, z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$ 07 3:3:1.2.1
- c. Derive one dimensional wave equation of the form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ 07 3:3:1.2.1

MODULE – 4

7. a. Compute a real root of $x \log_{10} x - 1.2 = 0$ by the method of false 06 3:4:1.2.1
position. Carry out three iterations.
- b. Find the interpolating polynomial for $f(x)$ using Newton’s forward 07 1:4:1.2.1
interpolation formula, given $f(0) = 0, f(2) = 4, f(4) = 56,$
 $f(6) = 204, f(8) = 496, f(10) = 980$ and hence find $f(3)$
- c. Using Newton’s divided difference formula to find $f(8), f(15)$ 07 1:4:1.2.1
from the following data.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

OR

8. a. Using Lagrange’s interpolation formula compute u_4 , given 06 3:4:1.2.1
 $u_0 = 707, u_2 = 819, u_3 = 866, u_6 = 966.$
- b. Use Simpson’s 1/3 rd rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking 6 sub- 07 1:4:1.2.1
intervals.
- c. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ taking 6 equal intervals using Trapezoidal rule. 07 3:4:1.2.1

MODULE – 5

9. a. Use Taylor’s series method to find $y(4.1)$ given that $\frac{dy}{dx} = \frac{1}{x^2+y}$ and 06 3:5:1.2.1
 $y(4) = 4$
- b. Given $\frac{dy}{dx} = 1 + \frac{y}{x}, y=2$ at $x=1$, find y at $x=1.2$ taking $h=0.2$ by 07 1:5:1.2.1
applying modified Euler’s method.
- c. Use Runge-Kutta method of fourth order, find $y(0.2)$ for the 07 3:5:1.2.1
equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0)=1$ taking $h=0.2$

OR

10. a. Employ Taylor's series method to find y at $x=0.1$ given that 06 3:5:1.2.1
$$\frac{dy}{dx} - 2y = 3e^x, y(0)=0$$
- b. Apply Milne's predictor-corrector formula to find $y(0.4)$ using the values $y(0)=1, y(0.1)=1.1113, y(0.2)=1.2507, y(0.3)=1.426$, given that $\frac{dy}{dx} = x^2 + y^2$. Write the answer approximating correct to four decimal places. 07 3:5:1.2.1
- c. Using Scilab develop a program to solve ODE using modified Euler's method for $f = y - 2x^2 + 1$ with the conditions $x_0 = 0, y_0 = 5, x_m = 1, h = 0.1$ 07 3:5:1.2.7

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