BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | |

Course Code | 2 | 1 | M | C | S | 3 | 1 |

Third Semester B.E. Degree Examinations, April/May 2023

FOURIER TRANSFORM, NUMERICAL METHODS & DISCRETE MATHEMATICS

(Common to CSE & AIML)

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	**a.**	Find the Fourier Transform of $f(x)=\begin{cases}1 & \text{for } \lvert x\rvert\le a\\ 0 & \text{for } \lvert x\rvert > a\end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$.	06	(2 :1 : 1.2.1)
	b.	Find the Fourier Transform of $f(x)=e^{-\lvert x\rvert}$.	07	(2 :1 : 1.2.1)
	c.	If $f(x)=\begin{cases}1\text{-x}^2 & ,\lvert x\rvert<1\\ 0 & ,\lvert x\rvert\ge 1\end{cases}$ find the Fourier Transform of $f(x)$ and hence find the value of $\int_0^\infty \frac{x\cos x-\sin x}{x^3}dx$.	07	(2 :1 : 1.2.1)
		OR		
2.	**a.**	Find the Fourier Sine Transform of $f(x)=e^{-\lvert x\rvert}$ and hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}dx, m>0$.	06	(2 :1 : 1.2.1)
	b.	Find the Fourier Sine and Cosine Transform of $f(x)=\begin{cases}\text{x} & 0<\text{x}<2\\ 0 & \text{elsewhere}\end{cases}$	07	(2 :1 : 1.2.1)
	c.	Find the Fourier Sine transform of $\frac{e^{-ax}}{x}, a>0$.	07	(2 :1 : 1.2.1)
		MODULE – 2		
3.	**a.**	Use Taylor's series method to find y at x=0.1 considering terms up to the third degree given that $\frac{dy}{dx}=x^2+y^2$ and $y(0)=1$.	06	(2 :2 : 1.2.1)
	b.	Using modified Euler's method find y(0.1) correct to four decimal places solving the equation $\frac{dy}{dx}=x-y^2$, y(0)=1 taking h= 0.1	07	(2 :2 : 1.2.1)
	c.	Use Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx}=\frac{y-x}{y+x}$, y(0)=1 taking h=0.2.	07	(2 :2 : 1.2.1)
		OR		

4.	**a.**	Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at x=0.8 by applying Milne's method.	**06**	(2 :2 : 1.2.1)
	b.	Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ compute y(1.4) by applying Adams-Bashforth method given that	**07**	(2 :2 : 1.2.1)

x	1	1.1	1.2	1.3
y	1	0.996	0.986	0.972

	c.	Use Modified Euler's method to find y (20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y (20) =5 and h=0.2.	**07**	(2 :2 : 1.2.1)

MODULE – 3

5.	**a.**	Show that, for any propositions p and q, the compound proposition $p \wedge (\neg p \wedge q)$ is a contradiction.	**06**	(2 :3 : 1.2.1)
	b.	Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.	**07**	(2 :3 : 1.2.1)
	c.	Prove the compound proposition $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ is logical equivalence.	**07**	(2 :3 : 1.2.1)

OR

6.	**a.**	State the converse, inverse and contra positive of the following conditions: (i). If a quadrilateral is a parallelogram, then its diagonals bisect each other. (ii). If a real number x^2 is greater than zero , then x is not equal to zero (iii). If a triangle is not isosceles, then it is not equilateral.	**06**	(2 :3 : 1.2.1)
	b.	Test whether the following is a valid argument. If I drive to work, then I will arrive tired. I am not tired (when I arrive at work). $\therefore$ I do not drive to work	**07**	(2 :3 : 1.2.1)
	c.	Let $p(x): x^2 - 7x + 10$, $q(x): x^2 - 2x - 3$, $r(x): x < 0$. Determine the truth or falsity of the following statements when the universe U contains only the integers 2 and 5. (i) $\forall x, p(x) \to \neg r(x)$ (ii) $\forall x, q(x) \to r(x)$ (iii) $\exists x, p(x) \to r(x)$	**07**	(2 :3 : 1.2.1)

MODULE – 4

7.	**a.**	Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if a divides b. Represent the relation R as a matrix and draw its digraph.	**06**	(2 :4 : 1.2.1)
	b.	Let $A = \{a, b, c\}$, and R and S be relations on A whose matrices are as given below: $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}; M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ Find RoS, SoR, RoR, SoS and their matrices.	**07**	(2 :4 : 1.2.1)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

c. A relation R on a set $A=\{a,b,c,d\}$ is represented by the following matrix: **07** (2 :4 : 1.2.1)

$$M_R=\begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&1&1\\0&0&1&1\end{bmatrix}$$. Determine whether R is an equivalence relation.

OR

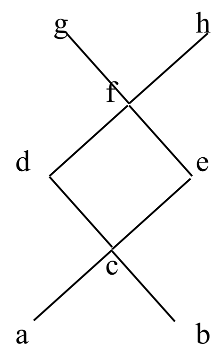
8. **a.** Let R is a relation on a non-empty set A defined as **06** (2 :4 : 1.2.1)

$R=\{(a,b): a\equiv b \bmod m\}$ or $R=\{(a,b): m \textit{ divides } (a-b)\}$.

Show that R is an equivalence relation.

b. Draw the Hasse diagram representing the positive divisors of 36 **07** (2 :4 : 1.2.1)

c. Consider the Hasse diagram of a poset (A,R) given below. **07** (2 :4 : 1.2.1)



If $B=\{c,d,e\}$, find (i) all upper bounds of B, (ii) all lower bounds of B, (iii) the least upper bound of B, (iv) the greatest lower bound of B.

MODULE – 5

9. **a.** Let f and g be functions from R to R defined by $f(x)=ax+b$ and $g(x)=1-x+x^2$. If $(gof)(x)=9x^2-9x+3$, determine a,b. **06** (2 :5 : 1.2.1)

b. Let $f: A\to B$ and $g: B\to C$ be any two functions. **07** (2 :5 : 1.2.1)

Then the following are true:

(i). If f and g are one-to-one, so is gof.

(ii). If f and g are onto, so is gof.

c. Given $p=\begin{pmatrix}1&2&3&4&5&6\\2&4&3&1&5&6\end{pmatrix}$, compute $p^{-1}, p^2,$ and p^3. **07** (2 :5 : 1.2.1)

OR

10. **a.** Solve the recurrence relation **06** (2 :5 : 1.2.1)

$F_{n+2}=F_{n+1}+F_n$ for $n\geq 0$

Given $F_0=0,\ F_1=1$

b. Solve the recurrence relation **07** (2 :5 : 1.2.1)

$2a_{n+3}=a_{n+2}+2a_{n+1}-a_n$ for $n\geq 0$ with $a_0=0, a_1=1, a_2=2$.

c. Solve the recurrence relation **07** (2 :5 : 1.2.1)

$a_n-6a_{n-1}+8a_{n-2}=9$ for $n\geq 2$, given that $a_0=10, a_1=25$.

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