

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN

--	--	--	--	--	--	--	--	--	--

Course Code

2	2	M	A	T	M	1	1
---	---	---	---	---	---	---	---

First Semester B.E. Degree Examinations, February 2024
MATHEMATICS FOR ME STREAM-I

Duration: 3 hrs

Max. Marks: 100

Note: 1. Answer any FIVE full questions choosing ONE full Question from each Module.
2. Missing data, if any, may be suitably assumed

<u>Q. No</u>	<u>Question</u>	<u>Marks</u>	<u>(RBTL:CO:PO)</u>
Module-1			
1. a.	Find the rank of the following matrix by row echelon form $\begin{pmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{pmatrix}$	06	(2 : 1 : 1.2.1)
b.	Test for consistency and solve the following system of equations. $5x + y + 3z = 20; 2x + 5y + 2z = 18; 3x + 2y + z = 14$	07	(3 : 1 : 1.2.1)
c.	Solve the following system of equations by Gauss elimination method $3x + 4y + 5z = 18; 2x - y + 8z = 13; 5x - 2y + 7z = 20$	07	(3 : 1 : 1.2.1)
OR			
2. a.	Solve the following system of equations by Gauss-Jordan elimination method $2x + y + 3z = 1; 4x + 4y + 7z = 1; 2x + 5y + 9z = 3$	06	(3 : 1 : 1.2.1)
b.	Solve the following system of equations using Gauss Seidel method $20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25$	07	(3 : 1 : 1.2.1)
c.	Find the numerically largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$. Perform 5 iterations.	07	(2 : 1 : 1.2.1)
Module-2			
3. a.	Prove with usual notation $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$	06	(3 : 2 : 1.2.1)
b.	Show that the following pairs of curves intersect each other orthogonally $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.	07	(2 : 2 : 1.2.1)
c.	Find the pedal equation of the curve $r(1 - \cos\theta) = 2a$.	07	(2 : 2 : 1.2.1)

OR

4. a. Prove with usual notation: $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$ 06 (3 :2 : 1.2.1)
- b. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$ 07 (2 :2 : 1.2.1)
- c. Write a program to Plot the cardioid $r = 3(1 + \cos\theta)$ 07 (1 :2 : 1.7.1)

Module-3

5. a. Using Maclaurin's series prove that 06 (3 :3 : 1.2.1)
- $$\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \dots\dots$$
- b. If $z = f(x + ct) + g(x - ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ 07 (3 :3 : 1.2.1)
- c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. 07 (3 :3 : 1.2.1)

OR

6. a. If $u = e^{-x}(x \cos y + y \sin y)$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 06 (3 :3 : 1.2.1)
- b. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$, where $u = x^2 + 3y^2 - z^3$, $v = 4x^2 yz$, $w = 2z^2 - xy$. 07 (2 :3 : 1.2.1)
- c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. 07 (2 :3 : 1.2.1)

Module-4

7. a. Solve: $x \frac{dy}{dx} + y = x^3 y^6$. 06 (3 :4 : 1.2.1)
- b. Solve: $(x^2 + y^3 + 6x)dx + y^2 x dy = 0$. 07 (3 :4 : 1.2.1)
- c. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$ 07 (2 :4 : 1.2.1)

OR

8. a. A series circuit with resistance R , inductance L and electromotive force E is governed by the differential equation $L \frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t 06 (2 :4 : 1.2.1)
- b. Solve: $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$ 07 (3 :4 : 1.2.1)

- c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation. Hence obtain the general and singular solution. **07** (3 : 4 : 1.2.1)

Module-5

9. a. Evaluate: $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$ **06** (2 : 5 : 1.2.1)
- b. Change the order of the integration and hence evaluate $\int_0^1 \int_{\sqrt{y}}^1 dx dy$. **07** (2 : 5 : 1.2.1)
- c. Find the area of the circle $x^2 + y^2 = a^2$ by double integration. **07** (2 : 5 : 1.2.1)

OR

10. a. Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0, x/a + y/b + z/c = 1$. **06** (2 : 5 : 1.2.1)
- b. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ **07** (3 : 5 : 1.2.1)
- c. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$ by expressing in terms of gamma functions. **07** (2 : 5 : 1.2.1)

** ** *