Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Subject Code** 2 1 M A T 1 1

First Semester B.E. Degree Examinations, May 2022

CALCULUS AND LINEAR ALGEBRA

(Common to all Branches)

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	**a.**	Prove that the Angle between tangent and radius vector for any curve at any point $p(r,\theta)$ is $\tan\phi = r\dfrac{d\theta}{dr}$.	06	(2 :1 : 1.1.1)
	b.	Prove that the curves $r^2 \sin 2\theta = a^2$ and $r^2 \cos 2\theta = a^2$ intersect orthogonally.	07	(2 :1 : 1.1.1)
	c.	For the curve Cardioid $r = a(1+\cos\theta)$ show that $\dfrac{\rho^2}{r}$ is a constant.	07	(2 :1 : 1.1.1)
		(OR)		
2.	**a.**	Obtain the centre of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at the point $(1,-1)$	06	(2 :1 : 1.1.1)
	b.	Obtain the pedal equation of the curve $\dfrac{2a}{r} = 1+\cos\theta$	07	(2 :1 : 1.1.1)
	c.	Obtain the radius of curvature for the curve $x^2 y = a(x^2+y^2)$ at $(-2a, 2a)$	07	(2 :1 : 1.1.1)
		MODULE - 2		
3.	**a.**	Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} - \cdots$	06	(2 :2 : 1.1.1)
	b.	If $u = f(x-y, y-z, z-x)$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$	07	(2 :2 : 1.1.1)
	c.	If $u = x+y+z,\ v = y+z,\ z = uvw$, find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$	07	(1 :2 : 1.1.1)
		(OR)		
4.	**a.**	Expand $e^{\sin x}$ in powers of x upto 4^{th} degree.	06	(1 :2 : 1.1.1)

(**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

	b.	If $z(x+y)=x^2+y^2$, Prove that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$	**07**	(2 :2 : 1.1.1)
	c.	A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum.	**07**	(2 :2 : 1.1.2)

MODULE-3

5.	**a.**	Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}dz\ dy\ dx$	**06**	(1 :3 : 1.1.1)
	b.	Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\ dy\ dx$ by changing the order of integration.	**07**	(1 :3 : 1.1.1)
	c.	Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}\ d\theta=\pi$	**07**	(2 :3 : 1.1.1)

(OR)

6.	**a.**	Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration.	**06**	(1 :3 : 1.1.1)
	b.	Evaluate: $\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}\ dx\ dy$ by changing to polar coordinates.	**07**	(1 :3 : 1.1.1)
	c.	Prove that $\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.	**07**	(2 :3 : 1.1.1)

MODULE-4

7.	**a.**	Solve $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$	**06**	(1 :4 : 1.1.1)
	b.	If the temperature of the air is 30^0 C and the metal ball cools from 100^0 C to 70^0 C in 15 minutes, find how long it will take for the metal ball to reach a temperature of 40^0 C.	**07**	(2 :4 : 1.1.2)
	c.	Solve the equation $(Px-y)(Px+y)=2P$ by reducing in to Clairaut's form, taking the substitution $X=x^2,\ Y=y^2$	**07**	(2 :4 : 1.1.1)

(OR)

8.	**a.**	Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$	**06**	(2 :4 : 1.1.1)
	b.	Find the orthogonal trajectories of $r^n=a^n\cos n\theta$	**07**	(1 :4 : 1.1.1)
	c.	A series circuit with resistance R, Inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt}+\mathrm{R}i=E$, where L and R are constants and initially the current i is zero .Find the current at any time t.	**07**	(2 :4 : 1.1.2)

MODULE-5

9. **a.** Find the rank of matrix $A=\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ by elementary row transformation. **06** (1 :5 : 1.1.1)

b. Solve the system of equations by gauss elimination method $x+y+z=9,\ x-2y+3z=8,\ 2x+y-z=3$ **07** (2 :5 : 1.1.1)

c. Investigate the values of λ and μ such that the system of equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ may have (a) Unique solution (b) Infinite solution (c) No solution. **07** (2 :5 : 1.1.1)

(OR)

10. **a.** Employ Gauss-Seidel iteration method to solve
$10x+y+z=12,\ x+10y+z=12,\ x+y+10z=12$.
Correct to three places of decimal. **06** (2 :5 : 1.1.1)

b. Find the largest Eigen value and Eigen vector of the matrix by Rayleigh's power method $A=\begin{bmatrix}25 & 1 & 2\\ 1 & 3 & 0\\ 2 & 0 & -4\end{bmatrix}$ **07** (1 :5 : 1.1.1)

c. Diagonalize the matrix $\begin{bmatrix}-19 & 7\\ -42 & 16\end{bmatrix}$ **07** (2 :5 : 1.1.1)

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