Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN											Course Code	2	1	M	A	T	2	1

Second Semester B.E. Degree Examinations, September/October 2022

ADVANCED CALCULUS AND NUMERICAL METHODS

(Common to all Branches)

Duration: 3 hrs **Max. Marks: 100**

Note: 1. Answer any **FIVE** *full questions, choosing* **ONE** *full question from each module.*
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	a.	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at $(2,-1,2)$.	06	(2 :1 : 1.1.1)
	b.	Find constants a and b such that $\vec{F}=(axy+z^3)i+(3x^2-z)j+(bxz^2-y)k$ is irrotational, and hence find the scalar potential.	07	(2 :1 : 1.1.1)
	c.	If $\vec{r}=xi+yj+zk$ and $r=\left\lvert\vec{r}\right\rvert$ then prove $\nabla^2(r^n)=n(n+1)r^{n-2}$	07	(3 :1 : 1.1.1)
		OR		
2.	a.	Find the directional derivative of $\Phi=x^2yz+4xz^2$ at $(1,-2,-1)$ along $2i-j-2k$.	06	(2 :1 : 1.1.1)
	b.	Evaluate $\int_c xydx+xy^2dy$ by Stoke's theorem where c is the square in the $x-y$ plane with vertices $(1,0),(-1,0),(0,1)$ and $(0,-1)$.	07	(2 :1 : 1.1.1)
	c.	Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ (or find the flux across the surface) given $\vec{F}=xi+yj+zk$ over the sphere $x^2+y^2+z^2=a^2$.	07	(3 :1 : 1.1.1)
		MODULE – 2		
3.	a.	Find $L\left[\frac{\cos 2t-\cos t}{t}\right]$	06	(2 :2 : 1.1.1)
	b.	Find $L[f(t)]$, if $f[t]=\begin{cases} t & 0\le t\le a \\ 2a-t & a\le t\le 2a \end{cases}$ where $f(t+2a)=f(t)$.	07	(2 :2 : 1.1.1)
	c.	Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ using convolution theorem.	07	(2 :2 : 1.1.1)
		OR		
4.	a.	Find $L^{-1}\left[\frac{s^2}{(s-1)(s+2)^2}\right]$.	06	(2 :2 : 1.1.1)

Note: **(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

	b.	Express $f[t]=\begin{cases}\cos t & 0\le t\le \pi \\ \text{sint} & t\succ \pi\end{cases}$ in terms of unit step function and hence find $L\left[f(t)\right]$,	07	(2 :2 : 1.1.1)
	c.	Using Laplace transformations solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=e^{-t}$, given $y(0)=0, y'(0)=0$.	07	(2 :2 : 1.1.1)

MODULE – 3

5.	**a.**	Solve $\left(4D^4-4D^3-23D^2+12D+36\right)y=0$.	06	(3 :3 : 1.1.1)
	b.	Solve $\left(D^2+4D\right)y=\sin 2x+x^2$.	07	(3 :3 : 1.1.1)
	c.	Using method of variation of parameter solve $\left(D^2+1\right)y=\frac{1}{1+\sin x}$	07	(3 :3 : 1.1.1)

OR

6.	**a.**	Solve $\left(D^3+D\right)y=e^{2x}$	06	(3 :3 : 1.1.1)
	b.	Solve $x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=\frac{1}{x}$	07	(3 :3 : 1.1.1)
	c.	Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\log(1+x)$	07	(3 :3 : 1.1.1)

MODULE – 4

7.	**a.**	Form a PDE of the family of all spheres whose centres lies on the plane $z=0$ and have a constant radius r.	06	(2 :4 : 1.1.1)
	b.	Solve $\frac{\partial^2 z}{\partial x\partial y}=\sin x\sin y$ for which $\frac{\partial z}{\partial y}=-2\sin y$ when $x=0$ and $z=0$, if y is an odd multiple of $\frac{\pi}{2}$.	07	(3 :4 : 1.1.1)
	c.	Derive one dimensional wave equation.	07	(3 :4 : 1.1.1)

OR

8.	**a.**	Form a PDE by eliminating arbitrary function $\Phi\left(x+y+z,x^2+y^2-z^2\right)=0$	06	(2 :4 : 1.1.1)
	b.	Solve $\frac{\partial^2 z}{\partial x^2}+z=0$ given that $x=0, z=e^y$ and $\frac{\partial z}{\partial y}=1$.	07	(3 :4 : 1.1.1)
	c.	Solve heat equation $u_t=c^2u_{xx}$ by method of separation of variables.	07	(3 :4 : 1.1.1)

MODULE – 5

9.	**a.**	Find the root of the equation $x\log(x)-1.2=0$ carry out three iterations using Newton-Raphson method.	06	(2 :5 : 1.1.1)
	b.	Given $\sin 45^\circ=0.7071, \sin 50^\circ=0.7660, \sin 55^\circ=0.8192, \sin 60^\circ=0.8660$, find $\sin 57^\circ$ using an appropriate interpolation formula.	07	(2 :5 : 1.1.1)

Note: **(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

c. Evaluate $\int_0^{\pi/2} \cos x \, dx$ by applying Simpson's 1/3rd rule taking seven ordinates. 07 (3 :5 : 1.1.1)

OR

10. a. Using secant method, solve $x^3 - 2x - 5 = 0$ carry up to three iterations. 06 (3 :5 : 1.1.1)

b. Use Lagrange's interpolation formula to find y at x=10 given 07 (2 :5 : 1.1.1)

x	5	6	9	11
y y	12	13	14	16

c. Using Simpson's 3/8th rule find $\int_0^1 \frac{x}{1+x^2} dx$ by taking seven ordinates and hence find the value of $\log 2$. 07 (2 :5 : 1.1.1)

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