Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | |

Course Code | 2 | 1 | M | A | T | 1 | 1 |

First Semester B.E. Degree Examinations, September/October 2022

CALCULUS AND LINEAR ALGEBRA

(Common to all Branches)

Duration: 3 hrs **Max. Marks: 100**

Note: 1. Answer any **FIVE** *full questions, choosing* **ONE** *full question from each module.*
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		MODULE – 1		
1.	a.	With usual notation, prove that $\tan\phi = r\left(\frac{d\theta}{dr}\right)$ with necessary figure.	06	(2 :1 : 1.1.1)
	b.	Show that the pairs of curves $r^n = a^n \cos n\theta$ and $r^n = a^n \sin n\theta$ intersect each other orthogonally.	07	(1 :1 : 1.1.1)
	c.	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	07	(1 :1 : 1.1.1)
		(OR)		
2.	a.	Obtain the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $(3a/2,\ 3a/2)$.	06	(2 :1 : 1.1.1)
	b.	Find the angle of intersection of the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$.	07	(1 :1 : 1.1.1)
	c.	Find the centre of curvature and circle of curvature for the curve $y^2 = 12x$ at $(3,6)$.	07	(1 :1 : 1.1.1)
		MODULE - 2		
3.	a.	Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$.	06	(2 :2 : 1.1.1)
	b.	If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	07	(2 :2 : 1.1.1)
	c.	Find the maximum and minimum values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	07	(1 :2 : 1.1.1)
		(OR)		
4.	a.	Expand $\log(\sec x)$ by Maclaurin's series up to the term containing x^4.	06	(2 :2 : 1.1.1)

(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)

	b.	A rectangular box open at the top is to have a volume of 32 cubic feet. Find its dimensions, if the total surface area is minimum.	07	(2 :2 : 1.1.2)
	c.	If $u=\frac{yz}{x},\ v=\frac{zx}{y},\ w=\frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$.	07	(1 :2 : 1.1.1)

MODULE-3

5.	a.	Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\ dz\ dy\ dx$.	06	(1 :3 : 1.1.1)
	b.	Evaluate by changing the order of integration $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy\ dy\ dx$.	07	(1 :3 : 1.1.1)
	c.	Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta=\pi$.	07	(2 :3 : 1.1.1)

(OR)

6.	a.	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\ dy$ by changing to polar coordinates.	06	(1 :3 : 1.1.1)
	b.	Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration.	07	(1 :3 : 1.1.1)
	c.	Prove that $\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.	07	(2 :3 : 1.1.1)

MODULE-4

7.	a.	Solve $(x^2+y^2+x)dx+xy\ dx=0$.	06	(1 :4 : 1.1.1)
	b.	Show that the family of parabolas $y^2=4a(x+a)$ is self orthogonal.	07	(2 :4 : 1.1.1)
	c.	If the temperature of the air is 30^0 C and the metal ball cools from 100^0 C to 70^0 C in 15 minutes, find how long it will take for the metal ball to reach a temperature of 40^0 C.	07	(2 :4 : 1.1.2)

(OR)

8.	a.	Solve $xy(1+xy^2)\frac{dy}{dx}=1$.	06	(1 :4 : 1.1.1)
	b.	A series circuit with resistance R, inductance L and e.m.f. E is governed by the differential equation $L\frac{di}{dt}+Ri=E$, where L and R are constants and initially the current $i=0$, find the current at any time t.	07	(2 :4 : 1.1.2)
	c.	Obtain the general solution and singular solution of the given Clairauts's equation $xp^3-yp^2+1=0$.	07	(2 :4 : 1.1.1)

(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)

MODULE-5

9.	a.	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix}$ by elementary row transformation.	06	(1 :5 : 1.1.1)
	b.	Solve the following system of equations by Gauss elimination method. $2x+y+4z=12,\ 4x+11y-z=33,\ 8x-3y+2z=20$.	07	(2 :5 : 1.1.1)
	c.	Find the largest Eigen value and the corresponding Eigen vector of the matrix $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by using the power method by taking initial vector as $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$.	07	(2 :5 : 1.1.1)

(OR)

10.	a.	Test for consistency and solve: $5x+y+3z=20,\ 2x+5y+2z=18,\ 3x+2y+z=14$	06	(2 :5 : 1.1.1)
	b.	Solve the following system of equations by Gauss-Seidel method. $5x+2y+z=12,\ x+4y+2z=15,\ x+2y+5z=20$ Carryout 4 iterations taking initial approximation as $(1,0,3)$	07	(1 :5 : 1.1.1)
	c.	Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$	07	(2 :5 : 1.1.1)

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(RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)