Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 2 | M | C | S | 3 | 1 |

Third Semester B.E. Degree Examinations, September 2024

GRAPH THEORY & DISCRETE MATHEMATICAL STRUCTURES

(Computer Science & Engineering)

Duration: 3 hrs **Max. Marks: 100**

Note:* *1. Answer any* FIVE *full questions choosing ONE Full Question from each Module
2. Formula Handbook is permitted
3. Missing data, if any, may be suitably assumed

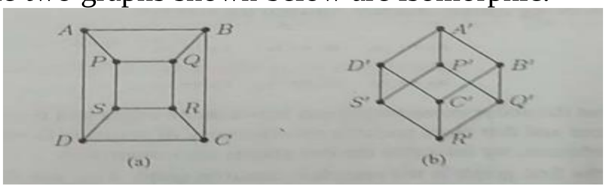
Q. No		Question	Marks	(RBTL:CO:PI)

Module-1

1. **a.** If $G = G(V, E)$ is a simple graph, prove that $2|E| \le |V|^2 - |V|$. **06** (2 :1 : 1.2.1)

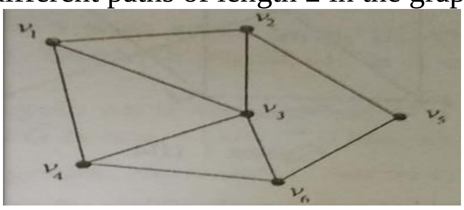
b. For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \le \frac{2m}{n} \le \Delta.$ **07** (2 :1 : 1.2.1)

c. Verify that the two graphs shown below are isomorphic. **07** (2 :1 : 1.2.1)



(OR)

2. **a.** Find the number of different paths of length 2 in the graph shown below. **06** (2 :1 : 1.2.1)



b. Exhibit the following **07** (2 :1 : 1.2.1)

(i) A graph which has both an Euler circuit and Hamilton cycle.
(ii) A graph which has an Euler circuit but no Hamilton cycle.
(iii) A graph which has a Hamilton cycle but no Euler circuit.
(iv) A graph which has neither a Hamilton cycle nor an Euler circuit.

c. In the complete graph with n vertices, where n is an odd number ≥ 3, there are $(n-1)/2$ edge-disjoint Hamiltonian cycles. **07** (2 :1 : 1.2.1)

Module-2

3. **a.** Prove that: Any connected graph with n vertices and n-1 edges is a tree. **06** (2 :2 : 1.2.1)

b. Suppose that a tree T has N_1 vertices of degree 2, N_2 vertices of degree 2, N_3 vertices of degree 3,…. , N_k vertices of degree k Prove that $N_1 = 2 + N_3 + 2N_4 + 3N_5 + ... + (k-2) N_k$. **07** (2 :2 : 1.2.1)

c. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendant vertices in T. **07** (2 :2 : 1.2.1)

(OR)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

4. **a.** Construct the binary tree that represents the prefix code $P = \{00, 01, 101, 110, 111\}$. **06** (2 :2 : 1.2.1)

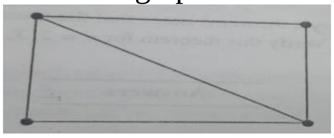
b. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **07** (2 :2 : 1.2.1)

c. Construct an optimal prefix code for the symbols with given weights in each of the following **07** (2 :2 : 1.2.1)

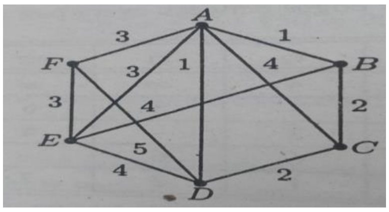
Symbols:	a	b	c	d	e	F	g
Weight:	10	30	5	15	20	15	5

Module-3

5. **a.** Find all the spanning trees of the graph shown below: **06** (2 :3 : 1.2.1)



b. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below. **07** (2 :3 : 1.2.1)

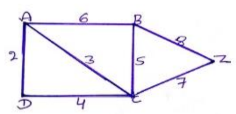


c. The following table gives the aerial distance (in Kms, rounded to nearest 50) between six cities A, B, C, D, E, F: Using Krushkal's method, find an air route shortest distance covering all the cities. **07** (2 :3 : 1.2.1)

	B	C	D	E	F
A	800	900	1800	700	650
B		650	1300	1350	1200
C			850	1650	1500
D				2500	2350
E					200

(OR)

6. **a.** For the network shown below, determine the maximum flow between A and Z by identifying a cut-set of minimum capacity. **06** (2 :3 : 1.2.1)

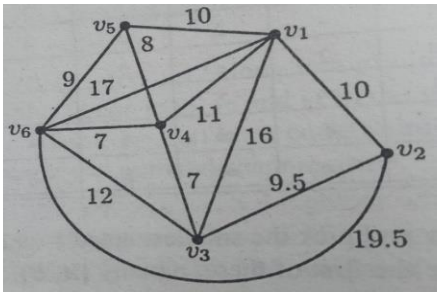


b. Eight cities A, B, C, D, E, F, G, H are required to be connected by a new railway network. The possible tracks and the cost of involved to lay them (in crores of rupees) are summarized in the following table: Determine a railway network of minimal cost that all connects all the cities. **07** (2 :3 : 1.2.1)

Track between	Cost	Track between	cost
A and B	155	D and F	100
A and D	145	E and F	150
A and G	120	F and G	140
B and C	145	F and H	150
C and D	150	G and H	160
C and E	95		

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

	c.	Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below.	**07**	(2 :3 : 1.2.1)



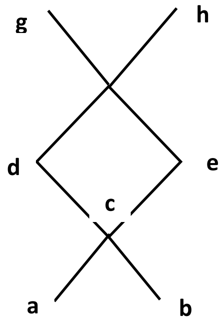
(OR)

7. **a.** Let $A=\{a,b,c,d\}$ and R be a relation on A that has the matrix — **06** — (2 :4 : 1.2.1)

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the digraph of R and list the in-degrees and out-degrees of all vertices.

b. Consider the poset whose Hasse diagram is shown below. — **07** — (2 :4 : 1.2.1)



If $B=\{c,d,e\}$, Find (i) all upper bounds of B (ii) all lower bounds of B (iii) the least upper bound of B (iv) the greatest lower bound of B

c. For a fixed integer n >1, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integers z. — **07** — (2 :4 : 1.2.1)

(OR)

8. **a.** Consider the functions f and g defined by $f(x)=x^3$ and $g(x)=x^2+1, \forall x \in R$. Find gof, fog, f^2 and g^2. — **006** — (2 :4 : 1.2.1)

b. Let $f: A \to B$ and $g: B \to C$ be any two functions. — **07** — (2 :4 : 1.2.1)
Then the following are true:
(i) If f and g are one-to-one, so is gof (ii) If gof is one-to-one, then f is one-to-one.

c. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$ (i) Write p as a product of disjoint cycles. (ii) Compute p^{-1} (iii) Compute p^2 and p^3 (iv) Find the smallest positive integer k such that $p^k = I_A$ — **07** — (2 :4 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

Module-5

9.	**a.**	The number of virus affected files in a system is 1000 (to start with) and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.	**06**	(2 :5 : 1.2.1)
	b.	Solve the recurrence relation $2a_n - 3a_{n-1} = 0$, for $n \geq 1$, given that $a_4 = 81$.	**07**	(2 :5 : 1.2.1)
	c.	Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$, for $n \geq 2$, given that $a_0 = 1, a_1 = 2$.	**07**	(2 :5 : 1.2.1)
		(OR)		
10	**a.**	Solve the Fibonacci relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given $F_0 = 0, F_1 = 1$.	**06**	(2 :5 : 1.2.1)
	b.	Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 4a_n = 8$, $n \geq 0$. given that $a_0 = 1$, $a_1 = 2$.	**07**	(2 :5 : 1.2.1)
	c.	Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2, n \geq 0$.	**07**	(2 :5 : 1.2.1)

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Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)