Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 2 | M | D | A | 3 | 1 |

Third Semester B.E. Degree Examinations, September 2024

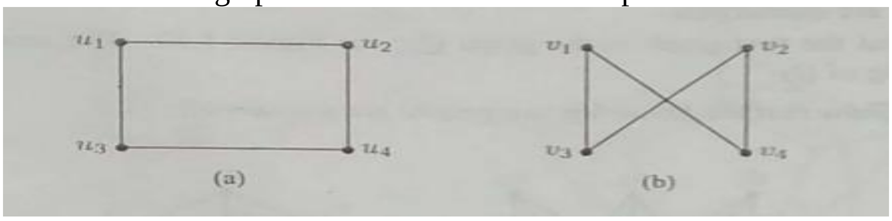
Graph Theory and Discrete Mathematical Structures, Probability and Statistics

AIML, CSE (AI) and CSE (DS)

Duration: 3 hrs **Max. Marks: 100**

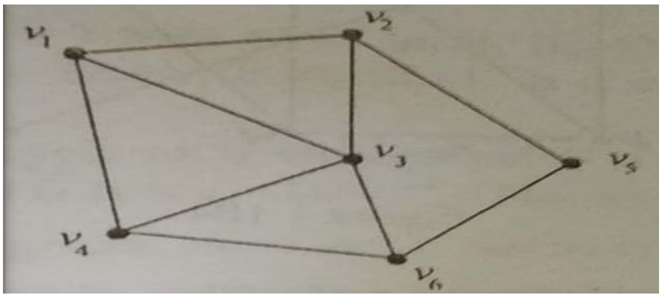
Note:* *1. Answer any* FIVE *full questions choosing ONE Full Question from each Module
2. Formula Handbook is permitted
3. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO:PI)
		Module-1		
1.	a.	If $G=G(V,E)$ is a simple graph, prove that $2\|E\| \le \|V\|^2-\|V\|$.	**06**	(2 :1 : 1.2.1)
	b.	Prove that in every graph, the number of vertices of odd degrees is even.	**07**	(2 :1 : 1.2.1)
	c.	Prove that the two graphs shown below are isomorphic.	**07**	(2 :1 : 1.2.1)

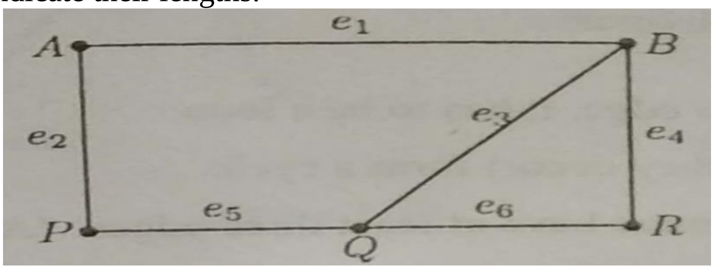


(OR)

2.	a.	Define (i) Simple Graph (ii) Complete Graph (iii) Bipartite Graph with examples.	**06**	(2 :1 : 1.2.1)
	b.	Determine the number of different paths of length 2 in the graph shown below	**07**	(2 :1 : 1.2.1)



	c.	Find all paths from vertex A to vertex R for the graph shown below. Also, indicate their lengths.	**07**	(2 :1 : 1.2.1)



Module-2

3.	a.	Let $A=\{1,2,3,4\}$ and let R be the relation on A defined by xRy if y=2x. (i) Write down R as set of ordered pairs. (ii) Draw the digraph of R. (iii) Determine the in-degrees and out-degrees of the vertices in the digraph.	**06**	(2 :2 : 1.2.1)
	b.	Define equivalence relation. For a fixed integer $n>1$, prove that the relation "congruent modulo" is an equivalence relation on the set of all integers z.	**07**	(2 :2 : 1.2.1)
	c.	Draw the Hasse diagram representing the positive divisors of 36.	**07**	(2 :2 : 1.2.1)

(OR)

4.	**a.**	Consider the functions f and g defined by $f(x)=x^3$ and $g(x)=x^2+1, \forall x \in R$. Find gof, fog, f^2 and g^2.	**06**	(2 :2 : 1.2.1)
	b.	Let $A=\{1,2,3,4,5,6,7,8\}$ and $p=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 5 & 1 & 8 & 7 & 6 \end{pmatrix}$. Express p as a product of disjoint cycles and compute p^{-1}.	**07**	(2 :2 : 1.2.1)
	c.	Let $f: A \to B, g: B \to C$ and $h: C \to D$ be three functions. Prove that $(hog)of = ho(gof)$.	**07**	(2 :2 : 1.2.1)

Module-3

5.	**a.**	Find a recurrence relation and the initial condition for the sequence 2, 10, 50, 250,… Hence find the general term of the sequence.	**06**	(2 :3 : 1.2.1)
	b.	Solve the recurrence relation $3a_{n+1}-4a_n=0$, for $n\geq 0$, given that $a_1=5$.	**07**	(2 :3 : 1.2.1)
	c.	Solve the recurrence relation $a_n=3a_{n-1}-2a_{n-2}=0$, for $n\geq 2$, given that $a_1=5$ and $a_2=3$.	**07**	(2 :3 : 1.2.1)

(OR)

6.	**a.**	Solve the recurrence relation $a_n-6a_{n-1}+9a_{n-2}=0$, for $n\geq 2$, given that $a_0=5$ and $a_1=12$.	**06**	(2 :3 : 1.2.1)
	b.	Solve the recurrence relation $F_{n+2}=F_{n+1}+F_n$ for $n\geq 0$, given $F_0=0, F_1=1$.	**07**	(2 :3 : 1.2.1)
	c.	Solve the recurrence relation $a_{n+2}+4a_{n+1}+4a_n=7, n\geq 0$. given that $a_0=1,\ a_1=2$.	**07**	(2 :3 : 1.2.1)

Module-4

7. **a.** Compute the coefficient of correlation and the equation of the lines of regression for the data, **06** (2 :4 : 1.2.1)

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

b. Show that if θ is the angle between the lines of regression, then $\tan\theta = \dfrac{\sigma_x\sigma_y}{\sigma_x{}^2+\sigma_y{}^2}\left(\dfrac{1-x^2}{r}\right)$. **07** (2 :4 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

c.	Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient	**07**	(2 :4 : 1.2.1)

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(OR)

8.	**a.**	Find the equation of the best fitting straight line $y = ax + b$ for the following data	**06**	(2 :4 : 1.2.1)

x	1	2	3	4	5
y	14	13	9	5	2

b.	Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at $x = 6$.	**07**	(2 :4 : 1.2.1)

x	1	2	3	4	5
y	10	12	13	16	19

c.	Fit a least square geometric curve $y = ax^b$ for the following data.	**07**	(2 :4 : 1.2.1)

x	1	2	3	4	5
y	14	13	9	5	2

Module-5

9.	**a.**	Find the value of k such that the following distribution represents finite probability distribution. Hence find its mean and standard deviation. Also find $P(x \le 1), P(x \succ 1), P(-1 < x \le 2)$.	**06**	(2 :5 : 1.2.1)

x	-3	-2	-1	0	1	2	3
$P(x)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

b.	Find the mean and standard deviation of Binomial distribution.	**07**	(2 :5 : 1.2.1)
c.	The probability that a pen manufactured by a factory be defective is $1/10$. If 12 such pens are manufactured, what is the probability that (i) exactly 2 are defective (ii) atleast 2 are defective (iii) none of them are defective	**07**	(2 :5 : 1.2.1)

(OR)

10	**a.**	Find mean and standard deviation of Poisson distribution.	**06**	(2 :5 : 1.2.1)
	b.	In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets.	**07**	(2 :5 : 1.2.1)
	c.	In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks normally distributed. Given $P(1.2263) = 0.39$ and $P(1.4757) = 0.43$.	**07**	(2 :5 : 1.2.1)

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Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)