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Course Code 

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**Fourth Semester B.E. Degree Examinations, Sept/Oct 2023**  
**LINEAR ALGEBRA, PROBABILITY AND STATISTICAL METHODS**  
(Common to CSE, AIML, AI, DS, ECE, EEE)

Duration: 3 hrs

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. Missing data, if any, may be suitably assumed

<u>Q. No</u>	<u>Question</u>	<u>Marks</u>	<u>(RBTL:CO: PI)</u>
<b><u>MODULE – 1</u></b>			
1.	<p>a. Define</p> <ol style="list-style-type: none"> <li>1. Vector space, Subspace.</li> <li>2. Basis and Dimension of the vector space</li> <li>3. Define linear transformation</li> </ol>	06	2:1:1.2.1
	<p>b. Find the dimension and a basis for the four fundamental subspaces for</p> $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$	07	2:1:1.2.1
	<p>c. <math>f: V_3(R) \rightarrow V_2(R)</math> is defined by <math>f(x, y, z) = (x + y, y + z)</math>. Show that f is a linear transformation.</p>	07	3:1:1.2.1
<b>OR</b>			
2.	<p>a. Consider the vector space <math>P(t)</math> with inner product <math>\langle f, g \rangle = \int_0^1 f(t)g(t)dt</math>. Apply the Gram-Schmidt orthogonalization process to the set <math>\{1, t, t^2\}</math> to find an orthogonal basis <math>\{f_0, f_1, f_2\}</math> with integer coefficients for <math>P_3(t)</math>.</p>	06	2:1:1.2.1
	<p>b. State and prove Rank-nullity theorem.</p>	07	2:1:1.2.1
	<p>c. Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of <math>R^4</math> spanned by <math>v_1 = (1, 1, 1, 1), v_2 = (1, 1, 2, 4), v_3 = (1, 2, -4, -3)</math>.</p>	07	3:1:1.2.1
<b><u>MODULE – 2</u></b>			
3.	<p>a. Find all the eigen values and the corresponding eigen vectors of the matrix <math>A = \begin{bmatrix} 8 &amp; -6 &amp; 2 \\ -6 &amp; 7 &amp; -4 \\ 2 &amp; -4 &amp; 3 \end{bmatrix}</math></p>	06	2:2:1.2.1
	<p>b. Diagonalize the matrix <math>A = \begin{bmatrix} 11 &amp; -4 &amp; -7 \\ 7 &amp; -2 &amp; -5 \\ 10 &amp; -4 &amp; -6 \end{bmatrix}</math></p>	07	2:2:1.2.1
	<p>c. Test to see if <math>A^T A</math> is positive definite <math>A = \begin{bmatrix} 1 &amp; 1 \\ 1 &amp; 2 \\ 2 &amp; 1 \end{bmatrix}</math></p>	07	2:2:1.2.1
<b>OR</b>			
4.	<p>a. Prove that, every square matrix can be uniquely expressed as sum of Hermitian and Skew Hermitian matrix</p>	06	2:2:1.2.1

b. Find the singular value decomposition of the matrix  $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$  07 2:2:1.2.1

c. Express the matrix  $A = \begin{bmatrix} 2 + 3i & 2 & 3i \\ -2i & 0 & 1 + 2i \\ 4 & 2 + 5i & -i \end{bmatrix}$  as the sum of a Hermitian and a Skew-Hermitian matrix. 07 2:2:1.2.1

### MODULE – 3

5. a. Compute the coefficient of correlation and the equation of the lines of regression for the data, 06 2:3:1.2.1

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

b. Show that if  $\theta$  is the angle between the lines of regression, then show that 07 3:3:2.2.1

$$\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left( \frac{1 - r^2}{r} \right)$$

c. Ten students got the following percentage of marks in two subjects  $x$  and  $y$ . Compute their rank correlation coefficient 07 2:3:1.2.1

Marks in x	78	36	98	25	75	82	90	62	65	39
marks in y	84	51	91	60	68	62	86	58	53	47

### OR

6. a. Fit a straight line  $y = ax + b$  for the following data by the method of least square. 06 2:3:1.2.1

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

b. Fit a parabola  $y = ax^2 + bx + c$  for the following data by the method of least square. 07 3:3:1.2.1

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

c. Fit a least square geometric curve  $y = ax^b$  for the following data. 07 2:3:1.2.1

x	1	2	3	4	5
y	14	13	9	5	2

### MODULE – 4

7. a. Find the value of  $k$  such that the following distribution represents finite probability distribution. Hence find its mean and standard deviation. also find  $P(x \leq 1)$ ,  $P(x > 1)$ ,  $P(-1 < x \leq 2)$  06 3:4:1.2.1

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

b. Find the mean and standard deviation of Binomial distribution. 07 3:4:1.2.1

c. The probability that a pen manufactured by a factory be defective is  $\frac{1}{10}$ . If 12 such pens are manufactured, what is the probability that  
 (i) exactly 2 are defective (ii) atleast 2 are defective  
 (iii) none of them are defective 07 3:4:1.2.1

**OR**

8. a. Find mean and standard deviation of Poisson distribution. 06 3:4:1.2.1
- b. In a certain factory turning out razor blades there is a small probability of  $1/500$  for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective, blades in a consignment of 10,000 packets. 07 3:4:1.2.1
- c. The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 to 75. 07 3:4:1.2.1  
( Use  $\phi(1) = 0.3413$  )

**MODULE – 5**

9. a. The joint distribution of two random variables X and Y is as follows. 06 3:5:1.2.1  
Compute the following  
(a)  $E(X)$  and  $E(Y)$  (b)  $E(XY)$  (c)  $\sigma_x$  and  $\sigma_y$  (d)  $COV(X, Y)$   
(e)  $\rho(X, Y)$

Y X \	-4	2	7
1	$1/8$	$1/4$	$1/8$
5	$1/4$	$1/8$	$1/8$

- b. Suppose X and Y are independent random variables with the following respective distribution, find the joint distribution of X and Y. also verify that  $COV(X, Y) = 0$ . 07 3:5:1.2.1

$x_i$	1	2
$f(x_i)$	0.7	0.3

$y_j$	-2	5	8
$g(y_j)$	0.3	0.5	0.2

- c. The joint probability distribution table for two discrete random variables X and Y is given by  $f(x, y) = k(2x + y)$  where x and y are the integers such that  $0 \leq x \leq 2, 0 \leq y \leq 3$ . 07 3:5:1.2.1  
a) Find the value of the constant k  
b) Find the marginal probability distributions of X and Y.  
c) Show that the random variables X and Y are dependent.

**OR**

10. a. Define the following 06 3:5:1.2.1  
a) Null Hypothesis  
b) Type-I and Type-II error  
c) Significance level
- b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.; 5,2,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure? 07 3:5:1.2.1  
(  $t_{0.05} = 2.201$  for 11 d.f )

- c. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4: 3: 2: 1 for the respective categories ( $\chi^2_{0.05} = 7.81$  for 3 d . f).

07 3:5:1.2.1

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MODEL QUESTION PAPER