BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT
(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** 2 1 M C M 4 1

Fourth Semester B.E. Degree Examinations, Sept/Oct 2023
COMPLEX ANALYSIS, PROBABILITY AND STATISTICAL METHODS
(Common to Civil & Mechanical)

Duration: 3 hrs **Max. Marks: 100**

Note: *1. Answer any* **FIVE** *full questions, choosing* **ONE** *full question from each module.*
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO: PI)
		MODULE – 1		
1.	**a.**	State & prove Cauchy – Riemann equation in Cartesian form.	06	2:1:1.2.1
	b.	Find the analytic function $\boldsymbol{f(z)}$ whose imaginary part is $e^x(x\, Siny + yCosy)$	07	2:1:1.2.1
	c.	Find the bilinear transformation which maps the points $z = 1,\ i, -1$ into the points $w = i,\ 0, -i$.	07	3:1:1.2.1
		OR		
2.	**a.**	State & prove Cauchy's integral theorem.	06	2:1:1.2.1
	b.	Evaluate $\int \frac{dz}{z^2-4}$ over (i) $C: \lvert z\rvert = 1$ (ii) $C: \lvert z\rvert = 3$	07	2:1:1.2.1
	c.	Expand $f(z) = \frac{dz}{z^2-4}$ over (i) $C: \lvert z\rvert = 1$ (ii) $C: \lvert z\rvert = 3$, $(iii) C: \lvert z + 2\rvert = 1$	07	3:1:1.2.1
		MODULE – 2		
3.	**a.**	A firm manufacturing two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs.3 on type B. Each product is processed on three machines M_1, M_2, M_3. Type A requires 4 minutes on machine M_1, 5 minutes on machine M_2, 6 minutes on on machine M_3 processing time. Where as type B requires 7 minutes on machine M_1, 8 minutes on machine M_2, 9 minutes on on machine M_3 processing time. The machine M_1 is available not more than 6 hours, while the machine M_2 and M_3 are available not more than 10 and 12 hours respectively during the working day. Formulate this problem as linear programming problem.	06	2:2:1.2.1
	b.	Use graphical method to minimize $Z = 20x_1 + 10x_2$ subject to the constraints, $x_1 + 2x_2 \le 40,\ 3x_1 + x_2 \ge 30,\ 4x_1 + 3x_2 \ge 60,\ x_1 \ge 0, x_2 \ge 0$.	07	2:2:1.2.1
	c.	Use simplex method to maximize $Z = x + 1.5y$ subject to the constraints, $x + 2y \le 160,\ 3x + 2y \le 240,\ x \ge 0,\ y \ge 0$.	07	2:2:1.2.1
		OR		
4.	**a.**	Define (i) Linear Programming Problem (ii) Feasible Solution (iii) Optimal Solution.	06	2:2:1.2.1

b. Use graphical method to maximize $Z = 5x + 6y$ subject to the constraints, $3x + 4y \leq 48,\ 9x + 4y \leq 72,\ x \geq 0,\ y \geq 0.$ — 07 — 2:2:1.2.1

c. Use simplex method to maximize $Z = 3x + 4y$ subject to the constraints, $2x + y \leq 40,\ 2x + 5y \leq 180,\ x \geq 0,\ y \geq 0.$ — 07 — 2:2:1.2.1

MODULE – 3

5. a. Compute the coefficient of correlation and the equation of the lines of regression for the data, — 06 — 2:3:1.7.1

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

b. Show that if θ is the angle between the lines of regression, then show that — 07 — 3:3:2.2.1

$$\tan\theta = \frac{\sigma_x\,\sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1 - r^2}{r}\right)$$

c. Ten students got the following percentage of marks in two subjects $x\ and\ y$. Compute their rank correlation coefficient — 07 — 2:3:1.2.1

Marks in x	78	36	98	25	75	82	90	62	65	39
marks in y	84	51	91	60	68	62	86	58	53	47

OR

6. a. Fit a straigh line $y = ax + b$ for the following data by the method of least square. — 06 — 2:3:1.2.1

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

b. Fit a parabola $y = ax^2 + bx + c$ for the following data by the method of least square. — 07 — 3:3:1.2.1

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

c. Fit a least square geometric curve $y = ax^b$ for the following data. — 07 — 2:3:1.2.1

x	1	2	3	4	5
y	14	13	9	5	2

MODULE – 4

7. a. Find the value of $\boldsymbol{k}$ such that the following distribution represents finite probability distribution. Hence find its mean and standard deviation. also find $P(x \leq 1),\ P(x > 1),\ P(-1 < x \leq 2)$ — 06 — 3:4:1.2.1

x	−3	−2	−1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

b. Find the mean and standard deviation of Binomial distribution. — 07 — 3:4:1.2.1

c. The probability that a pen manufactured by a factory be defective is $^{1}/_{10}$. If 12 such pens are manufactured, what is the probability that
(i) exactly 2 are defective (ii) atleast 2 are defective
(iii) none of them are defective — 07 — 3:4:1.2.1

OR

8. **a.** Find mean and standard deviation of Poisson distribution. 06 3:4:1.2.1

b. In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing (i)no defective (ii)one defective (iii)two defective, blades in a consignment of 10,000 packets. 07 3:4:1.2.1

c. The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will
be (i) less than 65 (ii) more than 75 (iii) between 65 to 75.
(use $\phi(1) = 0.3413$) 07 3:4:1.2.1

MODULE – 5

9. **a.** The joint distribution of two random variables X and Y is as follows. Compute the following 06 3:5:1.2.1
(a) E(X)and E(Y) (b)E(X Y) (c)σ_x and σ_y (d) COV(X , Y)
(e)ρ (X , Y)

X \ Y	−4	2	7
1	$1/8$	$1/4$	$1/8$
5	$1/4$	$1/8$	$1/8$

b. Suppose X and Y are independent random variables with the following respective distribution, find the joint distribution of X and Y. also verify that COV(X , Y) = 0. 07 3:5:1.2.1

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

c. The joint probability distribution table for two discrete random variables X and Y is given by $f(x, y) = k(2x + y)$ where x and y are the integers such that $0 \le x \le 2, 0 \le y \le 3$. 07 3:5:1.2.1
a) Find the value of the constant k
b) Find the marginal probability distributions of X and Y.
c) Show that tahe random variables X and Y are dependent.

OR

10. **a.** Define the following 06 3:5:1.2.1
a) Null Hypothesis
b) Type-I and Type-II error
c) Significance level

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.; 5,2,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure? 07 3:5:1.2.1
($t_{0.05} = 2.201$ *for* 11 *d.f*)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

c. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4: 3: 2: 1 for the respective categories (${\chi^2}_{0.05} = 7.81\ for\ 3\ d.f$). 07 3:5:1.2.1

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