BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** **2 1 M C M 3 1**

Third Semester B.E. Degree Examinations, March/April 2023

INTEGRAL TRANSFORMS & NUMERICAL METHODS

(Common to ME & CIVIL)

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO: PI)
		MODULE – 1		
1.	**a.**	State and prove Euler's formulae	**06**	(3:1:1.2.1)
	b.	Find the Fourier series of $f(x) = x(2\pi - x)$in $0 < x < 2\pi$	**07**	(2:1:1.2.1)
	c.	Find the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & in\ -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & in\quad 0 < x < \pi \end{cases}$	**07**	(2:1:1.2.1)
		OR		
2.	**a.**	Obtain the Fourier series to represent $f(x) = \|x\|$ in $(-l, l)$.	**06**	(2:1:1.2.1)
	b.	Obtain the Cosine half range Fourier series of $f(x) = x(\pi - x)$ in $0 < x < \pi$	**07**	(2:1:1.2.1)
	c.	The turning moment Ton the crank shaft of a steam engine for the crank angle θ is given as follows:	**07**	(3:1:1.2.1)

θ^0	0	30	60	90	120	150	180	210	240	270	300	330
T	0	2.7	5.2	7	8.1	8.3	7.9	6.8	5.5	4.1	2.6	1.2

Expand T as a Fourier series up to first harmonic.

Q. No		Question	Marks	(RBTL:CO: PI)
		MODULE – 2		
3.	**a.**	Find the Fourier Transform of $f(x) = e^{-\|x\|}$	**06**	(2:2:1.2.1)
	b.	Find the Complex Fourier transform of the function $f(x) = \begin{cases} 1 \ for & \|x\| \le a \\ 0\ for & \|x\| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$	**07**	(2:2:1.2.1)
	c.	If $f(x) = \begin{cases} 1 - x^2, & \|x\| < 1 \\ 0\quad, & \|x\| \ge 1 \end{cases}$ find the Fourier Transform of f(x) and hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx$	**07**	(2:2:1.2.1)
		OR		
4.	**a.**	Find the Fourier Sine and Cosine Transform of $f(x) = e^{-\alpha x}, \alpha > 0$	**06**	(2:2:1.2.1)

	b.	Find the Fourier Sine transform of $\frac{e^{-ax}}{x}, a > 0$	07	(2:2:1.2.1)
	c.	Find the Fourier Sine Transform of $f(x) = e^{-\lvert x\rvert}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2}\, dx,\ m > 0$	07	(2:2:1.2.1)
		MODULE – 3		
5.	a.	Use Taylor's method to find y at x=0.1 considering terms up to the third	06	(2:3:1.2.1)
	b.	Using Modified Euler's method find y (20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y (20) =5 and h=0.2.	07	(2:3:1.2.1)
	c.	Solve: $(y^2 - x^2)dx = (y^2 + x^2)dy$ for x=0.2 given that y=1 at x=0 by applying R-K method of order 4.	07	(2:3:1.2.1)
		OR		
6.	a.	Using modified Euler's method find y (0.1) correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2$, y (0) =1 taking h= 0.1	06	(2:3:1.2.1)
	b.	Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795,\ y(0.6) = 0.1762$. Compute y at x=0.8 by applying Milne's method	07	(2:3:1.2.1)
	c.	If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090$, find y (0.4) by using Adams-Bashforth method	07	(2:3:1.2.1)
		MODULE – 4		
7.	a.	Use Picard's method to find y (0.1) and z (0.1) given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ and $y(0) = 2, z(0) = 1$. (Carryout two approximations)	06	(2:4:1.2.1)
	b.	Use R-K method to solve system of equations; $\frac{dx}{dt} = y - t$, $\frac{dy}{dt} = x + t$, $x = 1, y = 1$ at $t = 0$. Compute $x(0.1)$and $y(0.1)$.	07	(3:4:1.2.1)
	c.	Obtain the Picard's third approximation to the solution of the system of equations $\frac{dx}{dt} = 2x + 3y,\ \frac{dy}{dt} = x - 3y;$ t=0, x=0, y=1/2. Hence find x and y at t=0.2	07	(3:4:1.2.1)
		OR		
8.	a.	By R-K method of fourth order, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x=0.2 correct to four decimal places, using the initial conditions y=1 and $y' = 0$ when x=0.	06	(2:4:1.2.1)
	b.	Apply Milne's method to compute y(1.4) given that $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ and the following table of initial values.	07	(3:4:1.2.1)

x	1	1.1	1.2	1.3
Y	2	2.2156	2.4649	2.7514
Y'	2	2.3178	2.6725	3.0657

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

	c.	Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$ using R-K method of fourth order	**07**	(3:4:1.2.1)

MODULE – 5

9.	**a.**	Obtain the Z-Transform of $\cos n\theta$ and $\sin n\theta$. Hence deduce the Z-Transform of (i) $k^n \cos n\theta$ (ii) $e^{-an} \sin n\theta$.	**06**	(2:5:1.2.1)
	b.	Find the Inverse Z-Transform of $\frac{z}{(z-1)(z-2)}$	**07**	(2:5:1.2.1)
	c.	Solve by using Z-Transforms: $y_{n+2} - 4y_n = 0$, given that $y_0 = 0, y_1 = 2$	**07**	(3:5:1.2.1)

OR

10.	**a.**	State and Prove Euler's Equation	**06**	(2:5:1.2.1)
	b.	Prove that the geodesics on a plane are straight line	**07**	(2:5:1.2.1)
	c.	A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary	**07**	(3:5:1.2.1)

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Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**