BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** 2 1 M E E 3 1

Third Semester B.E. Degree Examinations, March/April 2023
TRANSFORM CALCULUS & NUMERICAL METHODS
(Common to ECE & EEE)

Duration: 3 hrs **Max. Marks: 100**

Note: *1. Answer any* **FIVE** *full questions, choosing* **ONE** *full question from each module.*
2. Missing data, if any, may be suitably assumed

Q. No		*Question*	*Marks*	*(RBTL:CO: PI)*
		MODULE – 1		
1.	**a.**	State and prove Euler's formulae	**06**	(3:1:1.2.1)
	b.	Find the Fourier series in $(-\pi,\pi)$ to represent the function $f(x)=x-x^2$	**07**	(2:1:1.2.1)
	c.	An alternating current after passing through a rectifier has the form $I=\begin{cases} I_0\sin\theta & for\ \ 0<\theta\le\pi \\ 0 & for\ \ \pi<\theta\le 2\pi \end{cases}$ where I_0 is the maximum current. Express I as a Fourier series in $(0,2\pi)$.	**07**	(2:1:1.2.1)
		(OR)		
2.	**a.**	Obtain the Fourier series to represent $f(x)=1-x^2$, $-1\le x\le 1$	**06**	(2:1:1.2.1)
	b.	Obtain the Sine half range Fourier series of $f(x)=x^2$ in $0<x<\pi$	**07**	(2:1:1.2.1)
	c.	The following table gives the variations of a periodic current A over a period T. (table below) Find numerically the direct current part of the variable current and the amplitude of the first harmonic.	**07**	(3:1:1.2.1)

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp)	1.98	1.30	1.05	1.03	-0.88	-0.25	1.98

		MODULE – 2		
3.	**a.**	Find the Fourier Transform of $f(x)=e^{-\lvert x\rvert}$	**06**	(2:2:1.2.1)
	b.	Find the Complex Fourier transform of the function $f(x)=\begin{cases} 1 & for\ \ \lvert x\rvert\le a \\ 0 & for\ \ \lvert x\rvert>a \end{cases}$ and hence evaluate $\int_0^\infty \frac{sinx}{x}\,dx$	**07**	(2:2:1.2.1)
	c.	Find the Fourier Transform of $f(x)=\begin{cases} 1-\lvert x\rvert & for\ \ \lvert x\rvert\le 1 \\ 0 & for\ \ \lvert x\rvert>1 \end{cases}$ and hence deduce that $\int_0^\infty \frac{sin^2t}{t^2}\,dt=\frac{\pi}{2}$	**07**	(2:2:1.2.1)
		(OR)		

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

4. **a.** Find the Fourier Sine and Cosine Transform of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & elsewhere \end{cases}$ — **06** — (2:2:1.2.1)

b. Find the Fourier Sine transform of $\frac{e^{-ax}}{x}$ $, a > 0$ — **07** — (2:2:1.2.1)

c. Find the Fourier Sine Transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2}\, dx,\ m > 0$ — **07** — (2:2:1.2.1)

MODULE – 3

5. **a.** Obtain the Z-Transform of $\cos n\theta$ and $\sin n\theta$. Hence deduce the — **06** — (2:3:1.2.1)

b. Obtain the Z-Transform of $u_{n+1}\,, u_{n+2}$ from the basic definition. Also give the Z-transform of u_{n+k} — **07** — (2:3:1.2.1)

c. Using $Z_T(n^2) = \frac{z^2+z}{(z-1)^3}$ show that $Z_T((n+1)^2) = \frac{z^3+z^2}{(z-1)^3}$ — **07** — (2:3:1.2.1)

(OR)

6. **a.** Find the Inverse Z-Transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$ — **06** — (2:3:1.2.1)

b. Solve the Difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-Transform — **07** — (2:3:1.2.1)

c. For the signal $x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n]$, find the Z-Transform and Region of Convergence (ROC) — **07** — (2:3:1.2.1)

MODULE – 4

7. **a.** Use Taylor's series method to find y (4.1) given that $\frac{dy}{dx} = \frac{1}{x^2+y}$ and y (4) =4 — **06** — (2:4:1.2.1)

b. Using Modified Euler's method find y (20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y (20) =5 and h=0.2. — **07** — (3:4:1.2.1)

c. Use Runge-Kutta method of fourth order, find y (0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y (0) =1 taking h=0.2. — **07** — (3:4:1.2.1)

(OR)

8. **a.** Using modified Euler's method find y (0.1) correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2$, y (0) =1 taking h= 0.1 — **06** — (2:4:1.2.1)

b. Apply Milne's method to compute y (1.4) correct to 4 decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data: $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$ — **07** — (3:4:1.2.1)

c. Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ compute y (1.4) by applying Adams-Bashforth method given that — **07** — (3:4:1.2.1)

x	1	1.1	1.2	1.3
y	1	0.996	0.986	0.972

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

MODULE – 5

9.	**a.**	Use Picard's method to find y (0.1) and z (0.1) given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ and $y(0) = 2, z(0) = 1$. (Carryout two approximations)	**06**	(2:5:1.2.1)
	b.	Solve $\frac{dy}{dx} = 1 + zx$, $\frac{dz}{dx} + xy = 0$, $y(0) = 0, z(0) = 1$ at $x = 0.3$ by applying fourth order R-K method	**07**	(2:5:1.2.1)
	c.	Obtain the Picard's third approximation to the solution of the system of equations $\frac{dx}{dt} = 2x + 3y$, $\frac{dy}{dt} = x - 3y$; t=0, x=0, y=1/2. Hence find x and y at t=0.2	**07**	(3:5:1.2.1)

(OR)

10.	**a.**	By R-K method of fourth order, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x=0.2 correct to four decimal places, using the initial conditions y=1 and $y' = 0$ when x=0.	**06**	(2:5:1.2.1)
	b.	Apply Milne's method to compute y (0.8) given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values.	**07**	(2:5:1.2.1)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

	c.	Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$ using R-K method of fourth order.	**07**	(3:5:1.2.1)

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