BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 1 | M | C | S | 3 | 1 |

Third Semester B.E. Degree Examinations, March/April 2023
Fourier Transform, Numerical Methods and Discrete Mathematical Structures
(Common to CSE & AIML)

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO: PI)
		MODULE – 1		
1.	**a.**	Find the Fourier Transform of $f(x) = e^{-\|x\|}$	**06**	(2:1:1.2.1)
	b.	Find the Complex Fourier transform of the function $f(x) = \begin{cases} 1 \;\; for \;\; \|x\| \le a \\ 0 \; for \;\; \|x\| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{sinx}{x}\, dx$	**07**	(2:1:1.2.1)
	c.	If $f(x) = \begin{cases} 1 - x^2, \|x\| < 1 \\ 0 \qquad , \; \|x\| \ge 1 \end{cases}$ find the Fourier Transform of f(x) and hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\, dx$	**07**	(3:1:1.2.1)
		(OR)		
2.	**a.**	Find the Fourier Sine and Cosine Transform of $f(x) = e^{-\alpha x}, \alpha > 0$	**06**	(2:1:1.2.1)
	b.	Find the Fourier Sine transform of $\frac{e^{-ax}}{x}, a > 0$	**07**	(2:1:1.2.1)
	c.	Find the Fourier Sine Transform of $f(x) = e^{-\|x\|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx,\ m > 0$	**07**	3:1:1.2.1)
		MODULE – 2		
3.	**a.**	Use Taylor's method to find y at x=0.1 considering terms up to the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$	**06**	(2:2:1.2.1)
	b.	Using Modified Euler's method find y (20.2) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20)=5 and h=0.2.	**07**	(2:2:1.2.1)
	c.	Solve: $(y^2 - x^2)dx = (y^2 + x^2)dy$ for x=0.2 given that y=1 at x=0 by applying R-K method of order 4.	**07**	(2:2:1.2.1)
		(OR)		
4.	**a.**	Using modified Euler's method find y (0.1) correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2$, y (0) =1 taking h= 0.1	**06**	(2:2:1.2.1)
	b.	Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762$. Compute y at x=0.8 by applying Milne's method.	**07**	(2:2:1.2.1)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

	c.	If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090$, find y (0.4) by using Adams-Bashforth method	**07**	(2:2:1.2.1)

MODULE – 3

5.	**a.**	Define tautology and Verify for any three propositions p , q, r the	**06**	(2:3:1.7.1)
	b.	Prove that: $[(p \to q) \wedge (r \to q)] \Leftrightarrow [(p \vee r) \to q]$	**07**	(3:3:1.7.1)
	c.	State the converse, inverse and contra positive of the following conditions: 1. If a quadrilateral is a parallelogram, then its diagonals bisect each other. 2. If a real number x^2 is greater than zero, then x is not equal to zero. 3. If a triangle is not isosceles, then it is not equilateral.	**07**	(2:3:1.7.1)

(OR)

6.	**a.**	Define the dual of a logical statement, Write down the dual of the following propositions: (i). $p \to q$ (ii). $(p \to q) \to r$ (iii). $p \to (q \to r)$.	**06**	(2:3:1.7.1)
	b.	State whether the argument given below are valid or not. If an argument is valid, identify the tautology or tautologies on which it is based. (i). If Sachin hits a Century, he gets a free car. Sachin gets a free car. Therefore, Sachin hit a century. (ii). If I study, then I do not fail in the examination. If I do not fail in the examination, my father gifts a two-wheeler to me. Therefore, if I study then my father gifts a two –wheeler to me.	**07**	(3:3:1.7.1)
	c.	Let $p(x): x^2 - 7x + 10 = 0,\ q(x): x^2 - 2x - 3 = 0,\ r(x): x < 0.$ Determine the truth or falsity of the following statements when the universe U contains only the integers 2 and 5. (i). $\forall x,\ p(x) \to \neg r(x)$ (ii). $\forall x, q(x) \to r(x)$ (iii). $\exists x, q(x) \to r(x)$. (iv). $\exists x,\ p(x) \to r(x)$.	**07**	(2:3:1.7.1)

MODULE – 4

7.	**a.**	Let $A = \{1,2,3,4\}$ and let R be the relation on A defined by xRy if and only if “x divides y”, Write down the relation R as a set of ordered pairs, Draw the digraph of R and Determine the in-degree and out-degrees of the vertices in the digraph.	**06**	(3:4:1.7.1)
	b.	If $A = \{1,2,3,4\}$ and R, S are relations on A defines by $R = \{(1,2),(1,3)(2,4),(4,4)\}$ $S = \{(1,1),(1,2),(1,3),(1,4),(2,3),(2,4)\}$ find $R \circ S,\ \ S \circ R,\ \ R^2,\ \ S^2$, write down their matrices.	**07**	(3:4:1.7.1)
	c.	For a fixed integer $n \ \rangle\ 1$, prove that the relation “congruent modulo n” is equivalence relation on the set of all integers Z.	**07**	(3:4:1.7.1)

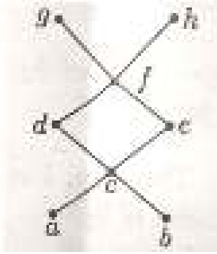
(OR)

8.	**a.**	Draw the Hasse diagram representing the positive divisor of 36.	**06**	(3:4:1.7.1)

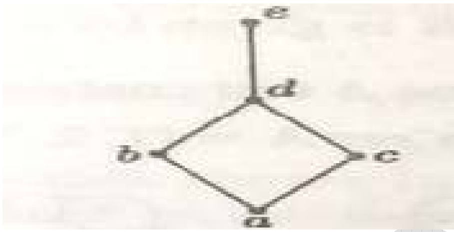
Note: (**RBTL - Revised Bloom’s Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

b. Consider the Hasse diagram of a poset (A, R) given below. If $B = \{c, d, e\}$, find (if they exists) **07** (3:4:1.7.1)

(i). all upper bounds of B (ii). all lower bounds of B.(iii). the least upper bound of B. (iv). The greatest lower bound of B.



c. For $A = \{a, b, c, d, e\}$,the Hasse diagram for the poset (A, R) is as shown below. **07** (3:4:1.7.1)



(a). Determine the relation R.
(b). Determine the relation matrix for R.
(c). Construct the diagraph for R.

MODULE – 5

9. a. Let $f : A \to B$ and $g : B \to C$ be any two functions, then show that the following are true: **06** (3:5:1.7.1)

(i). If f and g are one-to-one, so is gof. (ii). If gof is one-to-one, then f is one-to-one.

b. Let $A = B = R$, the set of all real numbers, and the function $f : A \to B$ and $g : B \to A$ be defined by **07** (3:5:1.7.1)

$$f(x) = 2x^3 - 1, \forall x \in A \; ; \; g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}, \forall y \in B.$$

Show that each of f and g is the inverse of the other.

$$f(x) = 2x^3 - 1, \forall x \in A \; ; \; g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}, \forall y \in B.$$

c. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A = \{1,2,3,4,5,6\}$, **07** (3:5:1.7.1)

(1). Write p as a product of disjoint cycles.

(2). Compute p^{-1} .

(3). Compute p^2 and p^3

(OR)

10. a. The number of viruses affected files in a system is 1000(to start with) and this increase 250% every two hours. Use a recurrence relation to determine the number of viruses affected in the system after one day. **06** (3:5:1.7.1)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

b. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}\ for\ n \geq 2,$ given that $a_1 = 5\ and\ \ a_2 = 3.$ **07** (3:5:1.7.1)

c. Solve the recurrence relation $a_n + 4a_{n-1} + 4a_{n-2} = 8, for\ \ n \geq 2,$ **07** (3:5:1.7.1)

and $a_0 = 1,\ \ a_1 = 2.$

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